## ( $\frac{3}{4}$ Multivariate Statistics of Tensor-Based Cortical Surface Morphometry

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## ABSTRACT

We propose a new set of multivariate statistics of tensorbased morphometric (TBM) framework that quantifies biological shape variations using the strain matrices. It can capture more deformation information than Jacobian matrix based local area element. with conformal slit map, we propose a constrained harmonic map based surface registration. We demonstrated the multivariate TBM outperformed other Jacobian matrix based statistics.

## Subjects

- 80 subjects had 1.5 T 3D T1-weighted brain MRIs - 40 healthy elderly subjects
- 40 subjects with genetically confirmed Williams syndrome
-Surfaces were obtained from a prior research (Thompson 2005)


## Methods

- Suppose ø: $S_{1} \rightarrow S_{2}$ is a map from the surface $S_{1}$ to the surface $S_{2}$. The derivative map of $\varnothing$ is the linear map between the tangent spaces, dø:TM(p) -> $T M(\varnothing(p))$, induced by the map $\varnothing$. In the local parameter domain, the derivative map is the Jacobian of $\varnothing$.
- In practice, smooth surfaces are usually approximated by triangle meshes. The map $\varnothing$ is approximated by a simplicial map, which maps vertices to vertices, edges to edges and faces to faces.
The derivative map dø is approximated by the linear map from one face $\left[v_{1}, v_{2}, v_{3}\right]$ to another one $\left[w_{1}, w_{2}, w_{3}\right]$. First, we isometrically embed the triangle $\left[v_{1}, v_{2}, v_{3}\right]$, [ $W_{1}, w_{2}, w_{3}$ ] onto the plane $\mathbf{R}^{2}$; the planar coordinates of the vertices of $v_{i}, w_{j}$ are denoted using the same symbols $v_{i}, W_{j}$. Then we explicitly compute the linear matrix for the derivative map $d \varnothing$,
$d \phi=\left[w_{3}-w_{1}, w_{2}-w_{1}\right]\left[v_{3}-v_{1}, v_{2}-v_{1}\right]^{-1}$
-We apply Hotelling's $T^{2}$ test on sets of values in the log-Euclidean space of the deformation tensors. Given two groups of $n$-dimensional vectors $S_{i}, i=1, \ldots, p, T_{j}$, ${ }_{j=1}, \ldots, q$, we use the Mahalanobis distance $M$ to measure the group mean difference,

$$
M=(\log \bar{S}-\log \bar{T}) \Sigma^{-1}(\log \bar{S}-\log \bar{T})
$$

Where $\bar{S}$ and $\bar{T}$ are the means of the two groups and $\Sigma$ is the combined covariance matrix of the two groups.


Comparison with Other Statistics

|  | Full <br> Matrix J | Determina <br> nt of J | Largest <br> EV of J | Pair of EV <br> of J |
| :--- | :---: | :---: | :---: | :---: |
| Left Cortex | $\mathbf{0 . 0 0 0 2}$ | 0.1933 | 0.1627 | 0.0003 |
| Right <br> Cortex | $\mathbf{0 . 0 0 0 1}$ | 0.1366 | 0.1201 | 0.0002 |

Permutation-based overall significance $p$ value for three experiments ( $J$ is the Jacobian matrix and EV stands for Eigenvalue). Multivariate TBM outperformed all other statistics.

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omputational Biolosy (CCB)

## Conclusions

- Proposed to apply Multivariate TBM to study cortical surface morphometry on WS data.
- Empirical results suggest that our method may outperform TBM methods based on simpler univariate surface measures.

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[^0]:    - Thompson, P, et. al (2005), 'Abnormal cortical compliexity and thickness profiles mapped in Williams syndrome', J. Neuroscience, pp. 41464158.

