Automatic Landmark Tracking and its Application to the Optimization of Brain Conformal Mapping

Lok Ming Lui, Yalin Wang, Tony F. Chan
Department of Mathematics
University of California, Los Angeles
$\{malmlui, ylwang, chan\}$@math.ucla.edu

Paul M. Thompson
Laboratory of Neuroimaging,
Department of Neurology
UCLA School of Medicine
thompson@loni.ucla.edu

Abstract

Anatomical features on cortical surfaces are usually represented by landmark curves, called sulci/gyri curves. These landmark curves are important information for neuroscientists to study brain diseases and to match different cortical surfaces. Manual labelling of these landmark curves is time-consuming, especially when there is a large set of data. In this paper, we proposed to trace the landmark curves on cortical surfaces automatically based on the principal directions. Suppose we are given the global conformal parametrization of a cortical surface, By fixing two endpoints, the anchor points, we propose to trace the landmark curves iteratively on the spherical/rectangular parameter domain along the principal direction. Consequently, the landmark curves can be mapped onto the cortical surface. To speed up the iterative scheme, a good initial guess of the landmark curve is necessary. We proposed a method to get a good initialization by extracting the high curvature region on the cortical surface using the Chan-Vese segmentation. This involves solving a PDE on the manifold using our global conformal parametrization technique. Experimental results show that the landmark curves detected by our algorithm closely resemble to those manually labelled curves. As an application, we used these automatically labelled landmark curves to build average cortical surfaces with an optimized brain conformal mapping method. Experimental results show our method can help automatically matching brain cortical surfaces.

1 Introduction

Finding feature points or curves on anatomical surfaces is an important problem in medical imaging. For example, anatomical features on the cortical surface can be represented by landmark curves, called sulci/gyri curves. These sulci/gyri curves are important information for neuroscientists to study brain diseases and to match different cortical surfaces. It is extremely time-consuming to label these landmark curves manually, especially when there is a large set of data. Therefore, an automatic or semi-automatic way to detect these feature curves is necessary. In this paper, we proposed to trace the landmark curves on the cortical surfaces automatically based on the principal directions. Suppose we are given the global conformal parametrization of a cortical surface, by fixing two endpoints, so-called the anchor points, we propose to trace the landmark curve iteratively on the spherical/rectangular parameter domain along one of the two principal directions. Consequently, the landmark curves can be mapped onto the cortical surface. To speed up the iterative scheme, a good initial guess of the landmark curve is necessary. Therefore, we proposed a method to get a good initialization by extracting the high curvature region on the cortical surface using the Chan-Vese segmentation [14]. This involves solving a PDE (Euler-Lagrange equation) on the manifold using the global conformal parametrization. As an application, we used these automatic labelled landmark curves to get an optimized brain conformal mapping, which can match important anatomical feautures across subjects.

Our paper is organized as follows: some previous works on the related topic will be studied in section 2. The basic mathematical theory will be discussed in section 3. In section 4, the algorithm of automatic landmark tracking and its application to the optimization of brain conformal mapping will be discussed. The experimental result will be studied in section 5. Finally, the conclusion and future works will be discussed in section 6.

2 Previous work

Automatic detection of sulci landmark on the brain has been widely studied by different research groups. Prince et al. [12] has proposed a method for automated segmentation of major cortical sulci on the outer brain boundary. It was based on a statistical shape model, which includes a network of deformable curves on the unit sphere, seeks geometric features such as high curvature regions, and labels such features via a deformation process that is confined within a spherical map of the outer brain boundary. Lohmann et al. [8] has proposed an algorithm that can automatically detect and attribute neuroanatomical names to the cortical folds using image analysis methods applied to magnetic resonance data of human brains. The sulci basins are segmented using a region growing approach. Zeng et al. [16] has proposed a method to automatic intrasulcal ribbon finding, by using the cortex segmentation with coupled surfaces via a level set method, where the outer cortical surface is embedded as the zero level set of a high-dimensional distance function. By using the distance function, they formulated the sulcal ribbon finding problem as one of surface deformations.

Optimization of surface diffeomorphisms by landmark matching has been studied intensively. Gu et al. [5] proposed to optimize the conformal parametrization by composing an optimal Mӧbius transformation so that it minimizes the landmark mismatch energy. The resulting parameterization remains conformal. Joan et al. [4] proposed to generate large deformation diffeomorphisms of the sphere onto itself, given the displacements of a finite set of template landmarks. The diffeomorphism obtained can match the geometric features significantly but it is, in general, not a conformal mapping. Leow et al. [7] proposed a level set based approach for matching different types of features, including points and 2D or 3D curves represented as implicit functions. Cortical surfaces were flattened to the unit square. Nine sulcal curves were chosen and were represented by the intersection of two level set functions, and used to constrain the warp of one cortical surface onto another. The resulting transformation was interpolated using a large deformation momentum formulation in the cortical parameter space, generalizing an elastic approach for cortical matching developed in Thompson et al. [13].

3 Basic mathematical theory

In this section, we will briefly review some basic mathematical theories.

Firstly, a diffeomorphism $f : M \to N$ is a *conformal mapping* if it preserves the first fundamental form up to a scaling factor (the conformal factor). Mathematically, this means that $ds_M^2 = \lambda f^*(ds_N^2)$, where ds_M^2 and ds_N^2 are the first fundamental form on M and N, respectively and λ is the conformal factor [11].

Next, we will give a brief overview of curvatures on a Riemann surface. The normal curvature κ_n of a Riemann surface in some direction is the reciprocal of the radius of the circle that best approximates a normal slice of surface in that direction, which varies with different directions. It follows:

$$\kappa_n = \mathbf{v}^T \mathbb{III} \mathbf{v} = \mathbf{v}^T \begin{pmatrix} e & f \\ f & g \end{pmatrix} \mathbf{v}$$

for any tangent vector $\mathbf{v}$. $\mathbb{III}$ is called the Weingarten matrix and is symmetric. Its eigenvalues and eigenvectors are called *principal curvatures* and *principal directions* respectively. The sum of the eigenvalues is called the *mean curvature*. The product of the eigenvalues is called the *Gaussian curvature*. The point on the Riemann surface at which the Weingarten matrix has the same eigenvalues is called the *umbilic point* [1].

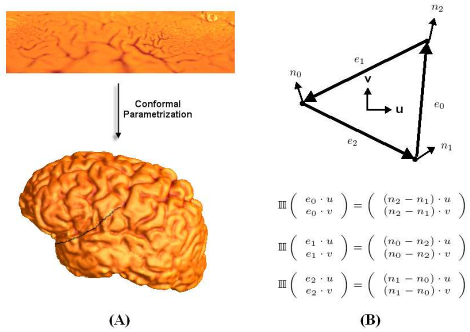

Figure 1. **LEFT** : Conformal parametrization of the cortical surface onto the 2D rectangle. **RIGHT** : A single face (triangle) of the triangulated mesh.

4 Algorithm

In this section, the algorithm of the automatic landmark tracking and its application to the optimization of brain conformal mapping will be discussed.

4.1 Computation of conformal parameterization

A diffeomorphism $f : M \to N$ is a *conformal mapping* if it preserves the first fundamental form up to a scaling factor (the conformal factor). Mathematically, this means that $ds_M^2 = \lambda f^*(ds_N^2)$, where ds_M^2 and ds_N^2 are the first fundamental form on M and N respectively and λ is the conformal factor [11]. For a diffeomorphism between two genus zero surfaces, a map is conformal if and only if it minimizes the harmonic energy,$E_{harmonic}$ [5]. However, this is not true for surfaces with genus equal to one or higher.

For high genus surfaces, Gu et. al [6] has proposed an efficient approach to parameterize surfaces conformally to the 2D rectangles. This approach is based on the homology group theory, the cohomology group theory and the Hodge theory.

We can then compute a conformal parametrization from the cortical surface onto the 2D domain. (See Figure 1(A)) [15]

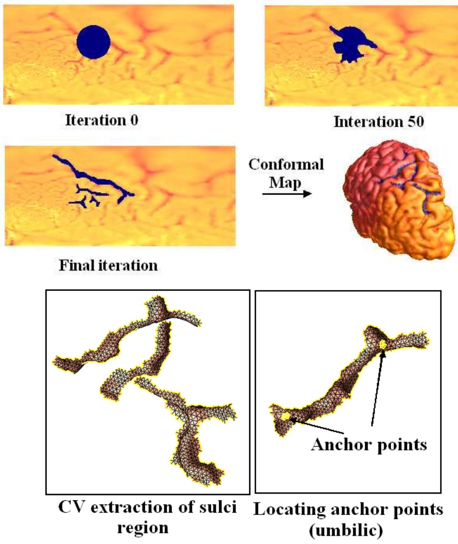

Figure 2. $\mathbf{TOP}$:Extraction of deep sulci region on the cortical surface by CV segmentation. $\mathbf{BOTTOM}$: After extracting the deep sulci region, we can then locate the umbilic points in each sulcal region, which are chosen as the anchor points

4.2 Computation of principal direction with global conformal parametrization

Denote the cortical surface by C. Let $\phi : D \to C$ be the global conformal parametrization of C where D is the rectangular parameter domain. Let λ be the conformal factor of ϕ. Following Rusinkiewicz's work [10], we can compute the principal directions, which are represented on the parameter domain D. This is based on the following three steps:

$\mathbf{Step1 : Per-Face\ Curvature\ Computation}$

The Weingarten matrix is defined in terms of the directional derivatives of the normal vector n:

$$\mathbb{III} = (D_u n, D_v n) = \begin{pmatrix} \frac{\partial n}{\partial u} \cdot u & \frac{\partial n}{\partial v} \cdot u \\ \frac{\partial n}{\partial u} \cdot v & \frac{\partial n}{\partial v} \cdot v \end{pmatrix}$$

where $u = \begin{pmatrix} \frac{1}{\sqrt{\lambda}} \\ 0 \end{pmatrix}$ and $v = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{\lambda}} \end{pmatrix}$ are the directions of an orthonormal coodinate system (represented on the parameter domain D) in the tangent plane.

Simple checking gives: $\mathbb{III}\mathbf{s} = D_{\mathbf{s}} n$, which is the derivative of the normal in the direction s and it is a vector on the tangent plane. Given a triangulation of the Riemann surface, we can approximate the Weingarten matrix $\mathbb{III}$ for each face (triangle).

For a triangle with three well defined directions (edges) together with the differences in normals in those directions (Refer to Figure 1 (B)). We have:

$$\mathbb{III}\begin{pmatrix} e_0 \cdot u \\ e_0 \cdot v \end{pmatrix} = \begin{pmatrix} (n_2 - n_1) \cdot u \\ (n_2 - n_1) \cdot v \end{pmatrix}; \mathbb{III}\begin{pmatrix} e_1 \cdot u \\ e_1 \cdot v \end{pmatrix} = \begin{pmatrix} (n_0 - n_2) \cdot u \\ (n_0 - n_2) \cdot v \end{pmatrix}$$

$$\mathbb{III}\begin{pmatrix} e_2 \cdot u \\ e_2 \cdot v \end{pmatrix} = \begin{pmatrix} (n_1 - n_0) \cdot u \\ (n_1 - n_0) \cdot v \end{pmatrix}$$

This gives a set of linear constraints on the elements of the Weingarten matrix, which can be determined using the least square method.

$\mathbf{Step2 : Coordinate\ system\ Transformation}$

After we have computed the Weingarten matrix on each face in the (u_f, v_f) coordinate system, we can average it with contribution from adjacent triangles. Suppose that each vertex p has its own orthonormal coordinate system (u_p, v_p). We have to transform the Weingarten matrix tensor into the vertex coordinates frame. The first component of $\mathbb{III}$, expressed in the (u_p, v_p) coordinate system, can be found as:

$$e_p = u_p^T \mathbb{III} u_p = (1,0)\begin{pmatrix} e_p & f_p \\ f_p & g_p \end{pmatrix}(1,0)^T$$

Thus,

$$e_p = (u_p \cdot u_f, u_p \cdot v_f)\mathbb{III}(u_p \cdot u_f, u_p \cdot v_f)^T$$

We can find f_p and g_p similarly.

$\mathbf{Step3 : Weighting}$

Another question is how much weighting do we need. That is, how much of the face curvature should be accumulated at each vertex. For each face f which is adjacent to the vertex p, we take the weighting $w_{f,p}$ to be the area of f divided by the squares of the lengths of the two edges that touch the vertex p.

4.3 Variational method for landmark tracking

Given the principal direction field $\overrightarrow{V}(t)$ with smaller eigenvalues on the cortical surface C. Fixing two anchor points (a & b) on the sulci, we propose a variational method to trace the sulci landmark curve iteratively. Let $\phi : D \to C$ be the conformal parametrization of C, $< \cdot, \cdot >$ to be its Riemannian metric and λ to be its conformal factor. We propose to locate a curve $\overrightarrow{c} : [0,1] \to C$ with endpoints a and b, which minimizes the following energy functional:

$$E_{principal}(\overrightarrow{c}) = \int_0^1 \left|\frac{\overrightarrow{c}\,'}{\sqrt{<\overrightarrow{c}\,', \overrightarrow{c}\,'>_M}} - \overrightarrow{V} \circ \overrightarrow{c}\right|_M^2 dt$$

$$= \int_0^1 \lambda\left|\frac{\overrightarrow{\gamma}\,'}{\sqrt{\lambda <\overrightarrow{\gamma}\,', \overrightarrow{\gamma}\,'>}} - \overrightarrow{V} \circ \overrightarrow{\gamma}\right|^2 dt = \int_0^1 \left|\frac{\overrightarrow{\gamma}\,'}{|\overrightarrow{\gamma}\,'|} - \sqrt{\lambda}\,\overrightarrow{V} \circ \overrightarrow{\gamma}\right|^2 dt$$

$$= \int_0^1 \left|\frac{\overrightarrow{\gamma}\,'}{|\overrightarrow{\gamma}\,'|} - \overrightarrow{G}(\overrightarrow{\gamma})\right|^2 dt$$

where $\overrightarrow{\gamma} = \overrightarrow{c} \circ \phi^{-1} : [0,1] \to D$ is the corresponding iterative curve on the parameter domain; $\overrightarrow{G}(\overrightarrow{\gamma}) = \sqrt{\lambda(\overrightarrow{\gamma})}\overrightarrow{V}(\overrightarrow{\gamma})$; $|\cdot|_M^2 =< \cdot, \cdot >_M$ and $|\cdot|$ is the (usual) length defined on D. By minimizing the energy E, we minimize the difference between the tangent vector field along the curve and the principal direction field $\overrightarrow{V}$. The resulting minimizing curve is the curve that is closest to the curve traced along the principal direction.

Let: $\overrightarrow{G} = (G_1, G_2, G_3)$; $\overrightarrow{K} = (K_1, K_2, K_3) = \frac{\overrightarrow{\gamma}'}{|\overrightarrow{\gamma}'|} - \overrightarrow{G}(\overrightarrow{\gamma})$

$\overrightarrow{L}_1 = \frac{(1,0,0)}{|\overrightarrow{\gamma}'|} - \frac{\gamma_1' \overrightarrow{\gamma}}{|\overrightarrow{\gamma}'|^3}$; $\overrightarrow{L}_2 = \frac{(0,1,0)}{|\overrightarrow{\gamma}'|} - \frac{\gamma_2' \overrightarrow{\gamma}}{|\overrightarrow{\gamma}'|^3}$; $\overrightarrow{L}_3 = \frac{(0,0,1)}{|\overrightarrow{\gamma}'|} - \frac{\gamma_3' \overrightarrow{\gamma}}{|\overrightarrow{\gamma}'|^3}$

Based on the Euler-Lagrange equation, we can locate the landmark curve iteratively using the steepest descent algorithm: $\frac{d\overrightarrow{\gamma}}{dt} = \Sigma_{i=1}^{3}[K_i \overrightarrow{L}_i]' + K_i \nabla G_i$ (**See Appendix**)

4.4 Landmark hypothesis by Chan-Vese Segmentation

In order to speed up the iterative scheme, a good initial guess of the landmark curve is necessary. Therefore, we proposed to get a good initialization by extracting the high curvature region on the cortical surface using the Chan-Vese (CV) segmentation [14]. We can extend the CV segmentation on $\mathbb{R}^2$ to any arbitrary Riemann surface M such as the cortical surface.

Let $\phi : \mathbb{R}^2 \to M$ be the conformal parametrization of the surface M.

We propose to minimize the following energy functional:

$$\begin{aligned}F(c_1, c_2, \psi) &= \int_M (u_0 - c_1)^2 H(\psi) dS + \int_M (u_0 - c_2)^2 (1 - H(\psi)) dS \\ &+ \nu \, \textbf{length of } \psi^{-1}(\{0\}) \\ &= \int_M (u_0 - c_1)^2 H(\psi) dS + \int_M (u_0 - c_2)^2 (1 - H(\psi)) dS \\ &+ \nu \int_M |\nabla_M H(\psi)|_M dS,\end{aligned}$$

where $\psi : M \to \mathbb{R}$ is the level set function and $|\cdot|_M = \sqrt{< \cdot, \cdot >}$.

With the conformal parametrization, we have:

$$\begin{aligned}F(c_1, c_2, \psi) &= \int_{\mathbb{R}^2} \lambda (u_0 \circ \phi - c_1)^2 H(\psi \circ \phi) dxdy \\ &+ \int_{\mathbb{R}^2} \lambda (u_0 \circ \phi - c_2)^2 (1 - H(\psi \circ \phi)) dxdy + \nu \int_{\mathbb{R}^2} \sqrt{\lambda} |\nabla H(\psi \circ \phi)| dxdy\end{aligned}$$

For simplicity, we let $\zeta = \psi \circ \phi$ and $w_0 = u_0 \circ \phi$. Fixing ζ, we must have:

$$c_1(t) = \frac{\int_\Omega w_0 H(\zeta(t,x,y)) \lambda dxdy}{\int_\Omega H(\zeta(t,x,y)) \lambda dxdy}$$

$$c_2(t) = \frac{\int_\Omega w_0 (1 - H(\zeta(t,x,y))) \lambda dxdy}{\int_\Omega (1 - H(\zeta(t,x,y))) \lambda dxdy}$$

Fixing c_1, c_2, the Euler-Lagrange equation becomes:

$$\frac{\partial \zeta}{\partial t} = \lambda \delta(\zeta) [\nu \frac{1}{\lambda} \nabla \cdot (\sqrt{\lambda} \frac{\nabla \zeta}{||\nabla \zeta||}) - (w_0 - c_1)^2 + (w_0 - c_2)^2]$$

Now, the sulci position on the cortical surface has relatively high curvature. Using CV segmentation, we can consider the intensity as the mean curvature. The sulci position on the cortical surface can then be located by extracting out the high curvature region. Fixing two anchor points inside the extracted region, we can get a good initialization of the landmark curve by looking for a shortest path inside the region that joins the two points. Also, we can consider the umbilic points inside the region as anchor points. By definition, the umbilic point on a manifold is the position at which the two principal curvatures are the same. Therefore, we can fix the anchor points inside the region by extracting region with small principal curvatures difference.

5 Optimization of brain conformal parametrization

One way to analyze and compare brain data from multiple subjects is to map them into a canonical space while retaining the original geometric information as far as possible. Surface-based approaches often map cortical surface data to a parameter domain such as a sphere, providing a common coordinate system for data integration [2, 3]. Any genus zero Riemann surfaces can be mapped conformally to a sphere, without angular distortion [5]. Therefore, conformal mapping offers a convenient way to parameterize the genus zero cortical surfaces of the brain. To compare cortical surfaces more effectively, it is desirable to adjust the conformal parameterizations to match specific anatomical features on the cortical surfaces as far as possible (such as sulcal/gyral landmarks in the form of landmark points or 3D curves lying in the surface). As an application of our automatic landmark tracking algorithm, we proposed to use the automatic labelled landmark curves to get an optimized brain conformal mapping. This matching of cortical patterns improves the alignment of data across subjects, e.g., when integrating functional imaging data across subjects, measuring brain changes, or making statistical comparisons in cortical anatomy. This is done by minimizing the compound energy functional $E_{new} = E_{harmonic} + \lambda E_{landmark}$, where $E_{harmonic}$ is the harmonic energy of the parameterization and $E_{landmark}$ is the landmark mismatch energy.

Suppose C_1 and C_2 are two cortical surfaces we want to compare. We let $f_1 : C_1 \to S^2$ be the conformal parameterization of C_1 mapping it onto S^2. Let $\{p_i : [0,1] \to S^2\}$ and $\{q_i : [0,1] \to S^2\}$ be the automatic labelled landmark curves, represented on the parameter domain S^2 with unit speed parametrization, for C_1 and C_2 respectively. Let $h : C_2 \to S^2$ be any homeomorphism from C_2 onto S^2. We define the landmark mismatch energy of h as, $E_{landmark}(h) = 1/2 \sum_{i=1}^{n} \int_0^1 ||h(q_i(t)) - f_1(p_i(t))||^2 dt$. where the norm represents distance on the sphere. By minimizing this energy functional, the Euclidean distance between the corresponding landmarks on the sphere is minimized [9].

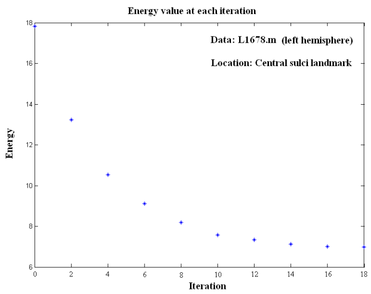

Figure 3. The figure shows the energy value $E_{principal}$ at each iteration.

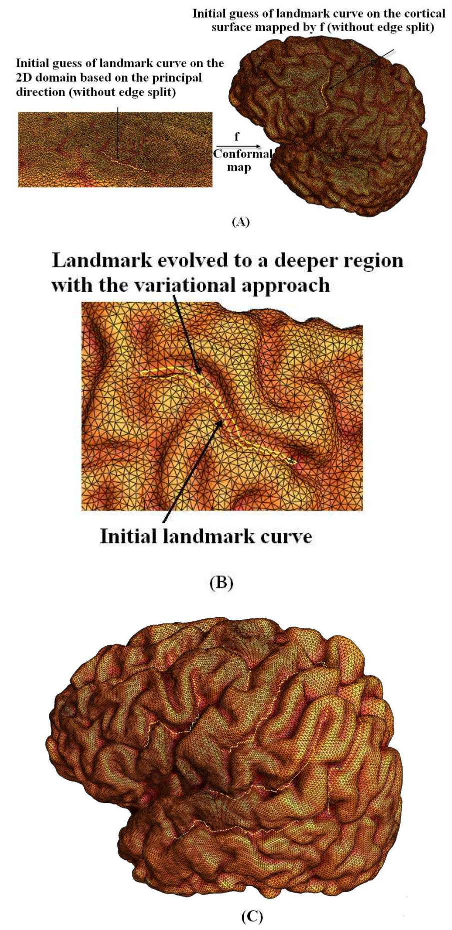

Figure 4. Automatic landmark tracking using a variational approach. In (A), we trace the landmark curves on the parameter domain along the edges whose directions are closest to the principal direction field. The corresponding landmark curves on the cortical surface is also shown. This gives a good initialization for our variational method to locate landmarks. (B) show how the initial landmark curve is evolved to a deeper sulci region with our iteration scheme. Note that the curve is evolved to a deeper region. In (C), 10 major sulci landmark curves are automatically traced with our algorithm.

Difference between the two landmarks is measured by:

$$E_{difference} = \int_0^1 ||\vec{c}_{principal}(t) - \vec{c}_{manual}(t)||^2 dt$$

L1627.m	Central	Pre-central	Post-central
Iteration 0	1.28	1.31	1.27
Iteration 30	0.21 ↓ 83.6%	0.22 ↓ 83.2%	0.21 ↓ 83.5%

L1680.m	Central	Pre-central	Post-central
Iteration 0	1.33	1.35	1.26
Iteration 30	0.24 ↓ 82.0%	0.28 ↓ 79.3%	0.26 ↓ 79.4%

Figure 5. Numerical comparison between automatic labelled landmarks and manually labelled landmarks by computing the Euclidean distance $E_{difference}$ (on the parameter domain) between the automatically and manually labelled landmark curves.

6 Experimental result

In our experiment, we tested our automatic landmark tracking algorithm on a set of left hemisphere cortical surfaces generated from brain MRI scans, scanned at 1.5 T (on a GE Signa scanner). In our experiments, 10 landmarks are automatically located on cortical surfaces. (Figure 4(C))

In Figure 2, we illustrate how we can effectively locate the initial landmark guess areas on the cortical surface using the Chan-Vese segmentation. Notice that the contour evolved to the deep sulci region.

Our variational method to locate landmark curve is illustrated in Figure 4. With the initial guess given by the Chan-Vese model (we choose two extreme points in the located area as the anchor points, see Figure 2 (BOTTOM)), we trace the landmark curves iteratively based on the principal direction field. In Figure 4 (a), we trace the landmark curves on the parameter domain along the edges whose directions are closest to the principal direction field. The corresponding landmark curves on the cortical surface is also shown in Figure 4 (a) (right). Figure 4 (b) show how the landmark curve is evolved to a deeper sulci region with our iterative scheme. Figure 3 shows the value of the energy functional at each iteration. Note that the energy decreases as the iteration increases. The energy value decreases by 62.33% after only 18 iterations. It means that our algorithm is very efficient.

In order to compare our automatic landmark tracing results with the manually labelled landmarks, we measures the Euclidean distance $E_{difference}$ (on the parameter domain) between the automatically and manually labelled landmark curves. Figure 5 shows the value of $E_{difference}$ at different iterations for different landmark curves. Note that the value becomes smaller as the iteration increases. It means that the automatically labelled landmark curves closely resemble to the manually labelled landmark curves as the iteration increases.

Figure 6 illustrates the application of our automatic land-

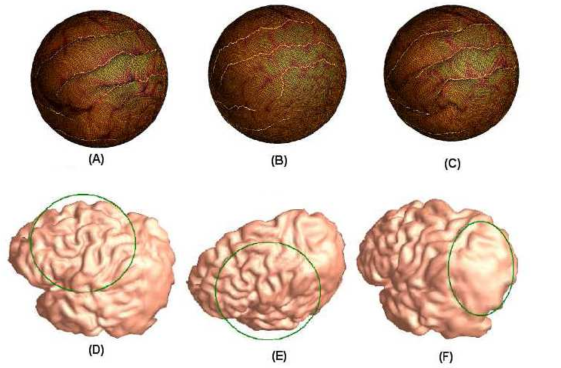

Figure 6. Optimization of brain conformal mapping using automatic landmark tracking. In (A) and (B), two different cortical surfaces are mapped conformally to the sphere. In (C), we map one of the cortical surface to the sphere using our algorithm. (D), (E), (F) shows the average map of the optimized conformal parametrization using the variational approach with the automatically traced landmark curves.

mark tracking algorithm. We optimize the brain surface our idea of the optimization of conformal mapping using the automatically traced landmark curves. Figure 6 (a) and (b) show two different cortical surfaces being mapped conformally to the sphere. Notice that the alignment of the sulci landmark curves are not consistent. In Figure 6 (c), the same cortical surface in (b) is mapped to the sphere using our method. Notice that the landmark curves closely resemble to those in (a), meaning that the alignment of the landmark curves are more consistent with our algorithm.

To visualize how well the automatic landmark location method can help the alignment of the important sulci landmarks is, we took average of the 15 optimized maps using the variational method [9]. Figure 6 (d,e,f) shows average maps at different angles. In (d) and (e), sulci landmarks are clearly preserved inside the green circle where landmarks are manually labelled. In (f), the sulci landmarks are averaged out inside the green circle where no landmarks are automatically detected. It means that our algorithm can help improving the alignment of the anatomical features.

7 Conclusion and future work

In this paper, we proposed to trace the landmark curves on the cortical surfaces automatically based on the principal directions. This is based on minimizing an energy functional. In order to speed up the iterative scheme, we proposed a method to get a good initialization by extracting the high curvature region on the cortical surface using the Chan-Vese segmentation. This involves solving a PDE on the manifold using our global conformal parametrization technique. Experimental results show that the landmark curves detected by our algorithm closely resemble to those manually labelled curves. Finally, we used the automatically labelled landmark curves to get an optimized brain conformal mapping, which can match important anatomical feautures across subjects. Experimental results show that the map we get can consistently align the important anatomical features. In the future, some numerical analysis of our algorithm will be done. A more quantitative analysis on how well our algorithm is will also be examined.

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