A 3D Volumetric Laplace-Beltrami Operator based Cortical Thickness Estimation Method

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Introduction

Alzheimer's disease (AD) is the most common form of cognitive disability in old people with the main feature as cortical atrophy. A key research question is how to quantitatively evaluate and compare grey matter thickness. Here we propose a heat kernel based method to estimate grey matter thickness. After constructing the cortical tetrahedral mesh, we adopt the heat kernel [1] based on volumetric Laplace-Beltrami operator proposed in our prior work [2] to calculate the cortical thickness.

Methods

First we fill the MRI space with the cubic background voxels with binvox software. Secondly, the cubic voxel containing the boundary surface and the internal voxel are split into the tetrahedrons using smoothing modules in software package CGAL [3]. The obtained tetrahedral mesh needs to be corrected to improve the quality and the smoothness based on harmonic function minimization [4]. Fig.1 shows an example of generated tetrahedral meshes and their tetrahedral element qualities. The figures from left to the right are the generated tetrahedral mesh (154,908 tetrahedrons) based on our method, the cross-section cut through the mesh according to y-axis, the dihedral angle histograms (the value ranges of the dihedral angle is within [11.92, 163.02]) and the tetrahedral element quality coefficients respectively. Then we define the tetrahedral mesh of the cortex as the finite solution space. After adding the contribution of the local stiffness matrix to global stiffness matrix, we can construct the discrete volumetric Laplace-Beltrami operator under the Dirichlet boundary condition [2]. Then we compute the heat kernel from the specific point on an isothermal surface to a different point on the next isothermal surface. According to the theory of the spectral analysis [5], the connection direction of the maximum transition probability is the direction of the temperature gradient. By repeating this process, a streamline of the cortex will be obtained by finding out the maximum heat transition probability between the isothermal surfaces. And the cortical thickness is estimated as the total length of the streamline. Our motivation is illustrated in Fig. 2. The heat diffusion is illustrated with spectrum (Fig.2 (a)) and the diffusion distance is illustrated in Fig. 2 (b).

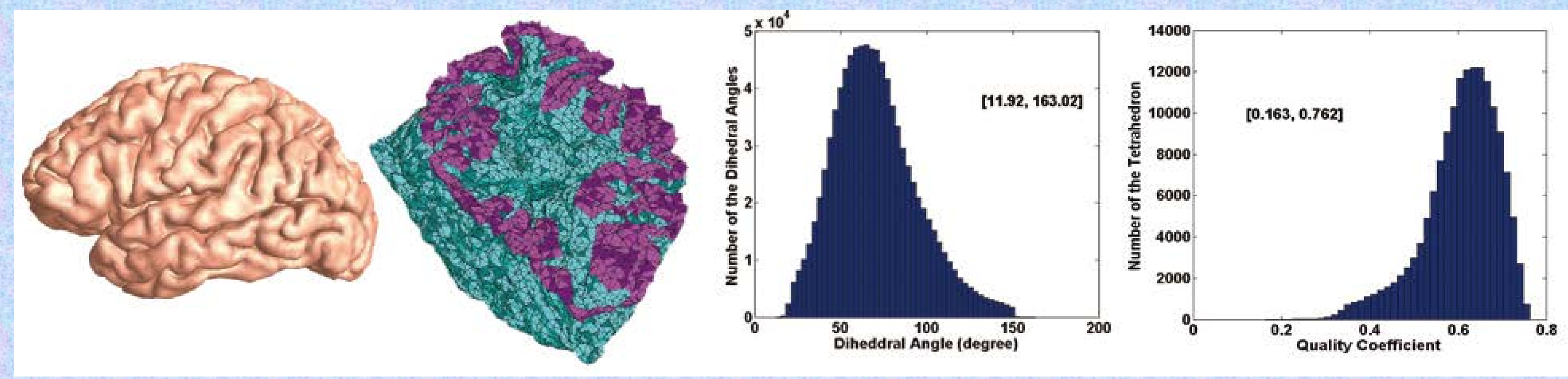

Fig.1 An example of generated tetrahedral meshes and their tetrahedral element qualities.

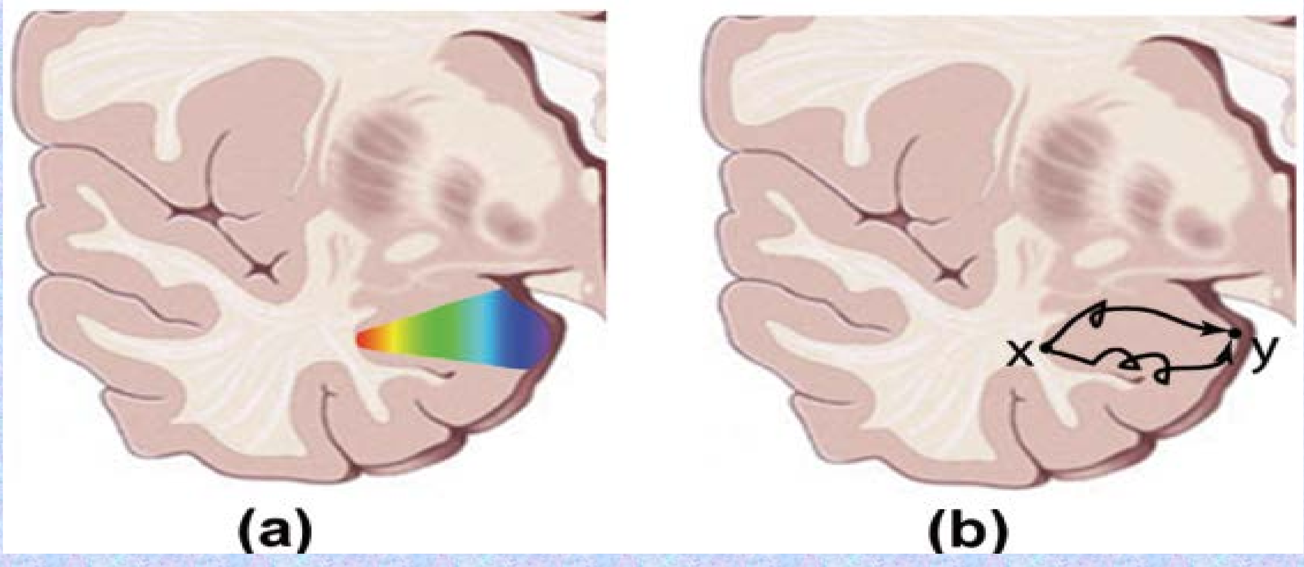

Fig.2 Illustration of heat diffusion on cortical structure

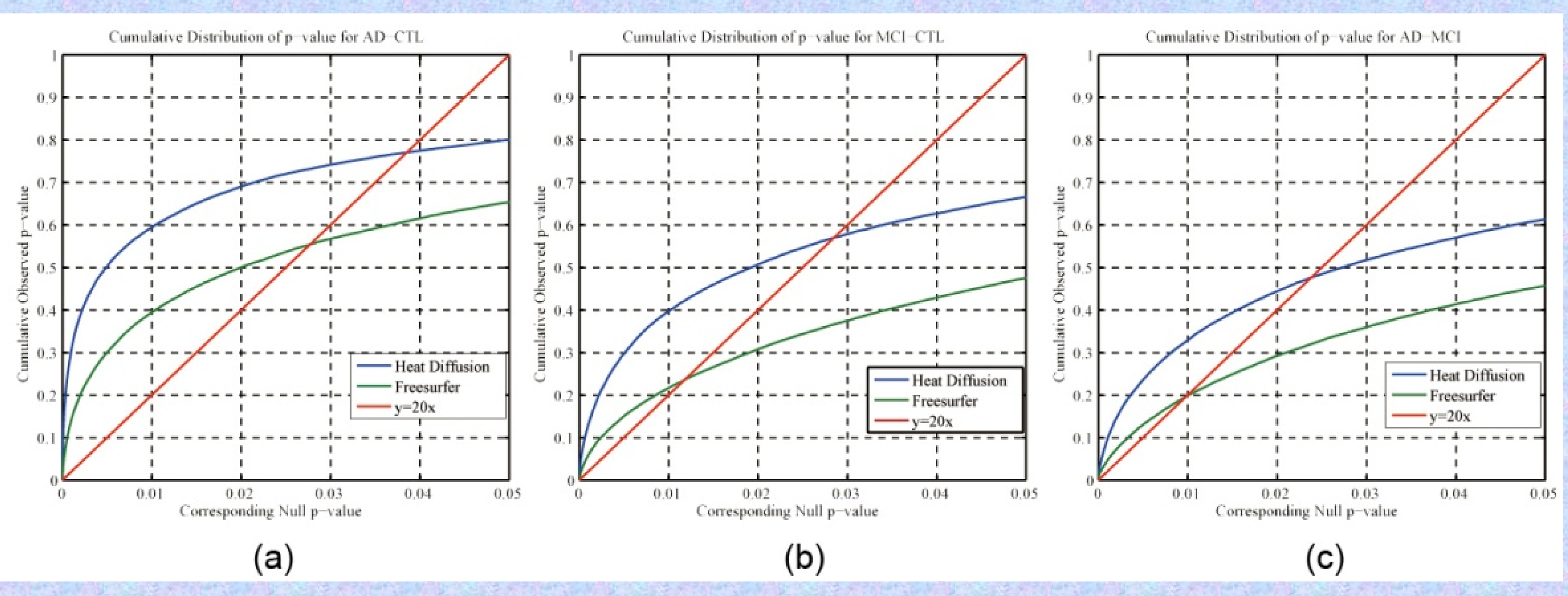

Fig.4 The cumulative distributions of *p*-values comparison for difference detected between the different groups

Results

Our dataset consists of 51 patients of AD, 45 patients of mild cognitive impairment (MCI) and 55 healthy controls. For comparison, the thicknesses estimated by our method and FreeSurfer [6] were linearly interpolated to the same surface template. In each case, the covariate (group membership) was permuted 5000 times and a null distribution was developed for the area of the average surface with group-difference statistics above the pre-defined threshold in the significance *p*-maps. Fig.3 shows the *p*-maps of group difference detected between AD and control ((a) and (d)), AD and MCI ((b) and (e)), control and MCI groups ((c) and (f)) and the significant level at each surface template point as 0.05. And (a), (b), (c) are results of FreeSurfer method, (d), (e), (f) are the results of our method. The non-blue color areas denote the statistically significant difference areas between two groups. All group difference *p*-maps were corrected for multiple comparisons using the widely-used false discovery rate method (FDR). Fig.4 (a)-(c) are the cumulative distribution function (CDF) plots showing the uncorrected *p*-values. As expected, we found very strong thickness differences between AD and control groups (*q*-value: 0.0385 with heat kernel method (Fig.3 (d)) and 0.0281 with FreeSurfer software (Fig.3 (a)), strong thickness differences between MCI and control groups (*q*-value: 0.0289 with heat kernel method (Fig.3 (e)) and 0.0133 with FreeSurfer software (Fig.3 (b)) and relatively less thickness differences between AD and MCI groups (*q*-value: 0.0247 with heat kernel method (Fig.3 (f)) and 0.0101 with FreeSurfer software (Fig.3 (c)).

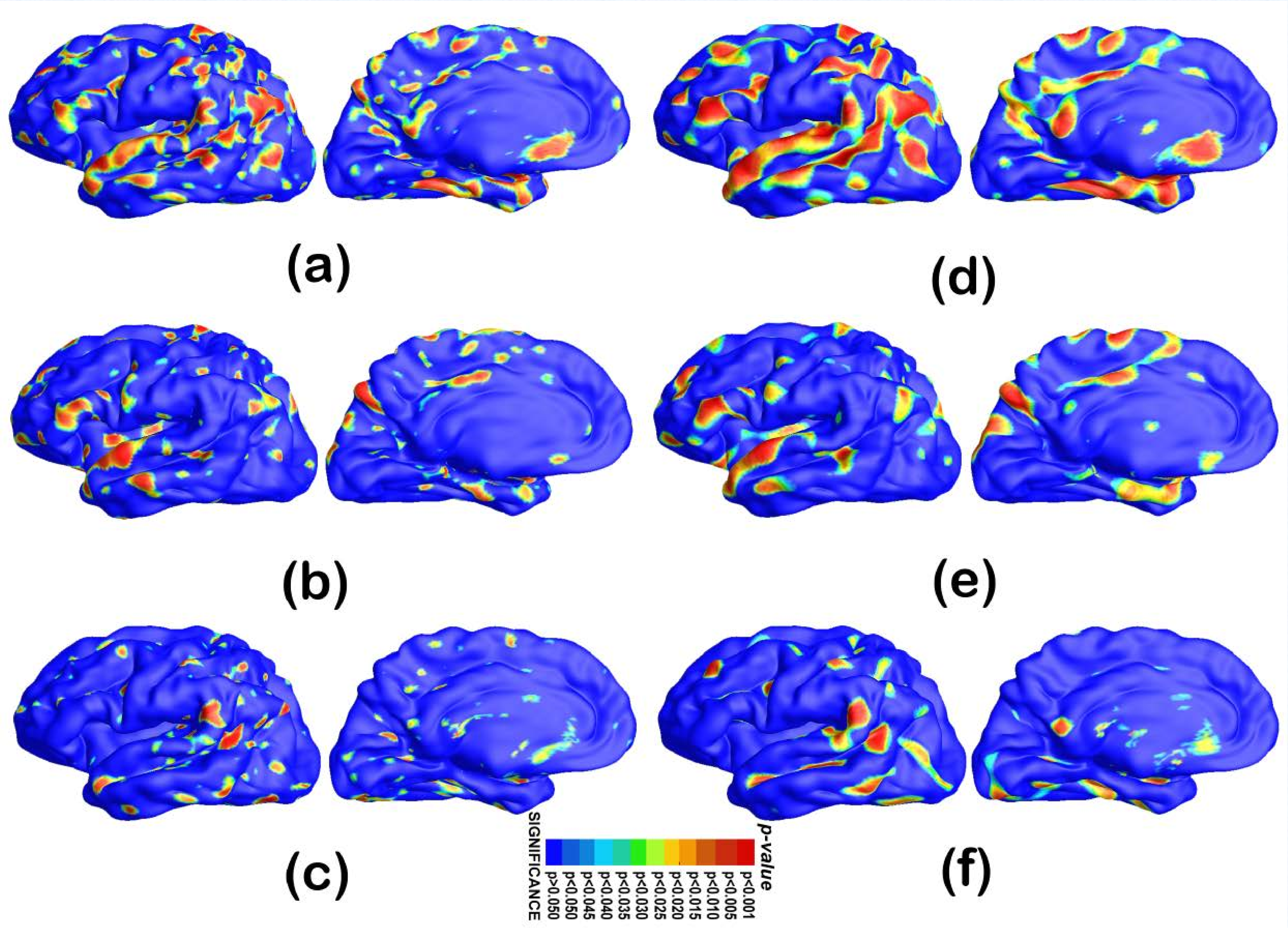

Fig.3 Statistical *p*-map results with the thickness measures of heat kernel diffusion (d-f) and FreeSurfer (a-c) show group differences.

Conclusions:

we present a heat kernel based thickness estimation algorithm which may improve the computational efficiency and accuracy for in vivo MR image cortical thickness estimation.

References:

[1] Hsu, E.P., 2002. Stochastic Analysis on Manifolds.American Mathematical Society.

[2] Wang, Y., Gu, X., Yau, S.T., 2004b. Volumetric harmonic map. Communications in Informationand Systems 3, 191–202.

[3] CGAL Editorial Board, 2013.Cgal, Computational Geometry Algorithms Library. www.cgal.org.

[4] Lederman, C., Joshi, A., Dinov, I., Vese, L., Toga, A., Van Horn, J.D., 2011. The generationof tetrahedral mesh models for neuroanatomical MRI. Neuroimage 55, 153–164.

[5] Coifman, R.R., Lafon, S., Lee, A.B., Maggioni, M., Nadler, B., Warner, F., Zucker, S.W.,2005a. Geometric diffusions as a tool for harmonic analysis and structure definition ofdata: diffusion maps. Proc. Natl. Acad. Sci. U.S.A. 102, 7426–7431.

[6] Fischl, B., Sereno, M.I., Dale, A.M., 1999a. Cortical surface-based analysis. II: Inflation,flattening, and a surface-based coordinate system. Neuroimage 9, 195–207.

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