SHAPE MATCHING WITH MEDIAL CURVES AND 1-D GROUP-WISE REGISTRATION

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ABSTRACT

We present a method for shape matching that approximates group-wise shape registration by reducing the problem to a 1-D registration. First, a novel medial curve method is proposed for computing a 1-D description of the shape. Second, a group-wise registration of the geometric descriptor is performed by directly minimizing the group variance. The resulting registration is used to adjust a global shape feature used to compute the final correspondence. Thus, the problems of description, registration, and statistical analysis are solved in one framework, while reducing the computational problem of group-wise registration of shapes substantially. We validate our method on 620 lateral ventricles extracted from the ADNI MRI dataset and 19 lateral ventricles from patients with HIV/AIDS and matched controls, also scanned with MRI. We show that our group-wise approach leads to improved statistical results, and also compare it to the SPHARM method.

Index Terms— skeletonization, medial curves, shape registration, group-wise registration, ADNI

1. INTRODUCTION

In computational anatomy, it is often desirable to locally compare the geometry of anatomical shapes. Essentially, anatomical shape analysis poses three related problems: (1) defining a meaningful and intuitive geometric descriptor, (2) registering the shapes in an unbiased manner, and (3) statistically comparing the resulting description. These three problems are interconnected, but existing methods generally solve these sequentially, or combine at best two of the three steps.

Several approaches have been proposed towards (1), including medial core-type methods [1,5], which define a local thickness of a shape, and an extension of Tensor Based Morphometry (TBM) to 2-manifolds [2]. The latter may require some imagination to be made intuitive, but it can be more statistically powerful for detecting correlations between subcortical shape and clinical, cognitive and CSF biomarkers.

In this work we choose a thickness-based approach based on a 1-dimensional single curve skeleton, as many anatomical shapes are oriented and therefore admit such a description. Thus, our geometric descriptor is the medial thickness, which is generally accepted as the most intuitive measure of shape morphometry. 1D medial representations in medical imaging were popularized by Stephen Pizer with the M-reps algorithm [5]. M-reps are comprised of a discrete web of "atoms," each of which describes position, width and local directions to the boundary, and an object angle between corresponding boundary points. However, the requirement that the direction between each atom's position and each corresponding surface point must be normal to the surface makes this method quite constrained even for anatomical shapes. To endow a shape with an M-reps description, the shape must either be quite simple, or it must be excessively smoothed. For example, such a description is not feasible for branching shapes such as the lateral ventricles, with all three horns prominently represented.

CM-reps, popularized by Paul Yushkevich and colleagues, is an extension of M-reps to the 2-D continuous medial core. CM-reps offer a way to derive boundaries from skeletons, by solving a Poisson-type partial differential equation with a nonlinear boundary condition [1]. The medial axis-anchored 3D parameterization of the shape-enclosed volumetric region is continuous, and allows a body-centered coordinate system for analyzing shape and appearance. However, because CM-reps do not reduce dimensionality, they do not produce significant computational speed-ups.

A number of methods have also been developed towards the registration step (2), such as Minimum Description Length (MDL) [7], intrinsic parameterization methods such as SPHARM [11] and conformal maps [2], direct shape mapping based on Laplace-Beltrami eignefunctions (LBE) [6], Spherical Demons (SD) [10], and q-maps [12]. MDL is an information-theoretic approach for group-wise shape registration. It leads to an unbiased correspondence within a sample, but it can be quite slow as it relies on simulated annealing, which may take many hours for just a handful of shapes. This method is not feasible for a large cohort such as ADNI where N is several hundred. SPHARM, conformal maps, and more recently q-maps compute optimal shape parameterizations, either by minimizing angle or area distortion, or by optimizing over a metric space of reparameterizations. These methods are quite general, but they do not exploit the shape descriptor, and thus do not couple steps (1) and (2). LBE-based direct maps generate feature functions to be used for direct registration using level set embeddings. Similarly, SD performs shape registration on the sphere by registering spherical images in the Diffeomorphic Demons framework. Thus, both SD and LBE are capable of combining description with registration. However, these methods are designed to register an image to either another image or, in the case of SD, to an atlas formed by a set of pre-registered images. This biases the resulting parameterization, as the statistical analysis step (3) remains uncoupled with the rest of the processing. Spherical Demons perhaps comes closest to coupling all 3 steps above. Further, LBE is only feasible for fairly simple shapes with a clear spatial orientation and no branching, a constraint similar to M-reps' limitations.

To mitigate this problem, we maintain the reduced 1-D representation with a single curve, but relax the M-reps' normal-to-

surface constraint. Instead, our medial representation is based on a variational framework in which the curve is defined by minimizing a weighted total distance. Thus, the geometry of the curve and the shape are tied more loosely, and more classes of shapes admit such a representation. Although a single curve representation may appear inferior to 2-D representations due to the ill-posed nature of the 1-D problem, in fact the reduced dimensionality is of great benefit for subsequent processing.

Our registration step combines many of the registration methods above. We use the spherical domain and a combination of area and angle- preserving regularization, while minimizing the L^2 distance between pairs of feature functions. To enable fast spherical registration, we modify the unconstrained spherical parameterization (USP) algorithm [8] by simply adding the L^2 fidelity terms to the cost function. The resulting registration remains nearly as fast as the original USP tool.

The feature functions are induced by our medial curve, in the spirit of LBE. In our case, however, the curve is computed first, while in LBE the curve is induced by the function. Finally, before registering the spherical images to a target, we perform a 1-D to 1-D group-wise registration in the spirit of MDL. Our 1-D registration non-linearly remaps the scalar feature to be used in spherical registration by minimizing the variance of the feature to be used as a geometric descriptor, i.e., the feature that will be compared statistically. Thus, this step combines all 3 shape comparison problems.

Our contribution is threefold: first, we develop a relaxed medial curve framework which allows noisy and branching shapes to be parameterized consistently by a single curve. This allows the computation of intuitive shape description by medial thickness and natural feature functions for registration. Second, as the curve has lower dimensionality, we enable a quasi-group-wise shape registration by group-wise registration of 1-D functions, thereby incorporating variance reduction and improved statistical sensitivity into the registration. And third, using a modification of the USP algorithm, we show that the spherical registration resulting from the 1-D registration step leads to improved statistical results based on ADNI and a dataset of lateral ventricles from HIV+ subjects and age-matched controls.

2. MEDIAL CURVE FRAMEWORK

Finding the curve-skeleton of an orientable surface is not a well-defined problem, but some properties are generally accepted as desirable [6]:

(1) *Centered*: we would like our curve to be "locally" in the middle of the shape. In medical imaging, numerical accuracy is vital when estimating local thickness on boundaries of shapes.
(2) *Onto, smooth mapping*: there must be a surjective, smooth mapping from the surface to the curve. This enables us to use the medial curve for registration.
(3) *Consistent geometry*: this property requires that the geometry of the curve depend continuously on the shape

We assume that our anatomical shape can be represented by a single curve. We also assume that the ends of the curve lie on the surface. With the topology of the curve fixed, we focus on property (1) above. Intuitively, we can say that a curve is the *medial* curve if it is smooth and every point on it is "locally in the middle" of the shape. Given a surface S, the curve $\boldsymbol{c}(t),\ t \in [0,1]$, should be a global minimum of $R(\boldsymbol{c},\boldsymbol{c}',S) =$

$$\int_0^1 \bar{\int_{\boldsymbol{p}\in S}} w(\boldsymbol{c}(t),\boldsymbol{c}'(\boldsymbol{t}),\boldsymbol{p},S)\|\boldsymbol{c}(t)-\ \boldsymbol{p}\|^2\, dSdt$$

$$\boldsymbol{c}(0)\in S, \boldsymbol{c}(1)\in S \qquad (\mathbf{1})$$

Here, $w(\boldsymbol{c},\boldsymbol{c}',\boldsymbol{p},S)$ is the weight defining "localness" of point $\boldsymbol{p}$ relative to $\boldsymbol{c(t)}$. A variety of weighting functions can be devised, one of the simplest ones being $w(\boldsymbol{c},\boldsymbol{c}',\boldsymbol{p},S) =$

$$\begin{cases} w_1(\boldsymbol{c}(t),\boldsymbol{c}'(\boldsymbol{t}),\boldsymbol{p},S), & if\ min\{D[\boldsymbol{g}(u,\boldsymbol{p},\boldsymbol{c}(t)),S]|u\in(0,1)\}>0 \\ 0\ , & otherwise \end{cases}$$

$$w_1(\boldsymbol{c},\boldsymbol{c}',\boldsymbol{p},S) = \delta_\alpha\left(\frac{\langle \boldsymbol{c}-\boldsymbol{p},\boldsymbol{c}'\rangle}{\|\boldsymbol{c}-\boldsymbol{p}\|\|\boldsymbol{c}'\|}\right), \qquad (\mathbf{2})$$

where $D(\boldsymbol{x},S)$ is the signed distance of S at $\boldsymbol{x}$, and $\boldsymbol{g}(u,\boldsymbol{a},\boldsymbol{b}) = \boldsymbol{a}+u(\boldsymbol{b}-\boldsymbol{a})$. Here, δ_α is a continuous approximation of the Dirac delta. We used $\delta_\alpha = \alpha e^{-x^2\alpha^2}$ in our implementation. This particular weighting function is only concerned with the differential properties of the curve, not those of the surface, making it quite stable when applied to noisy surfaces. The weighting function decreases continuously as the vector from the surface to the curve becomes less normal to the curve, and vanishes when there is no line of sight to the surface.

To ensure that the curve is smooth, we add a smoothing term, so that our final cost function takes the form:

$$L(\boldsymbol{c}(t),S) = R(\boldsymbol{c},\boldsymbol{c}',S) + \beta\int_0^1 \|\boldsymbol{c}''(t)\|^2\ dt \qquad (\mathbf{3})$$

Solving for the Euler-Lagrange equation of **(3)**, we derive the descent direction for the curve: $\frac{d\boldsymbol{c}}{d\tau} =$

$$\int_{\boldsymbol{p}\in S}^{-}\left[\left(\frac{d}{dt}\frac{dw}{d\boldsymbol{c}'}-\frac{dw}{d\boldsymbol{c}}\right)\|\boldsymbol{c}-\ \boldsymbol{p}\|^2 - w(\boldsymbol{c}-\ \boldsymbol{p})\right]dS - \beta\boldsymbol{c}^{(4)} \quad (\mathbf{4})$$

3. GROUP-WISE 1-D REGISTRATION

To enable curve-based registration for shapes described by a single curve, we derive two surface based functions. The first is the global orientation function (GOF), defined as

$$G(\boldsymbol{p}) = \arg\min_t\{\|\boldsymbol{c}(t)-\ \boldsymbol{p}\|, t\epsilon[0,1]\} \qquad (\mathbf{5})$$

This is similar to the first LB eigenfunction [6], except it can have more than two local extrema. Our geometric descriptor is then the familiar thickness measure:

$$D(\boldsymbol{p}) = \|\boldsymbol{c}(G(\boldsymbol{p}))-\ \boldsymbol{p}\| \qquad (\mathbf{6})$$

As we are interested in improving statistical sensitivity of medial thickness, we seek a set of mappings $r_n\colon[0,1]\to\mathbb{R}$, such that the total sample variance of a set of functions $f_n\colon[0,1]\to\mathbb{R}$ is reduced by minimizing the cost function

$$C(\{f_n,r_n\}_{n=1}^N) =$$

$$\int_0^1 \frac{1}{N}\sum_{n=1}^N f_n^2[t-r_n(t)] - \frac{1}{N^2}\left(\sum_{n=1}^N f_n[t-r_n(t)]\right)^2 + \frac{\sigma^2}{N}\sum_{n=1}^N[r_n'(t)]^2\ dt, \qquad [t-r_n(t)]\in[0,1]\ \forall n \quad (\mathbf{7})$$

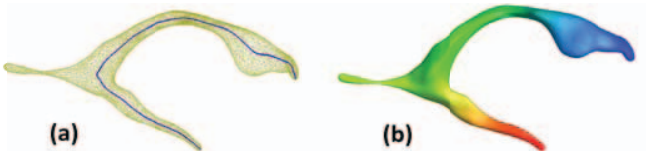

Figure 1. (a) Medial curve of a left lateral ventricular surface, based on the proposed framework; (b) the resulting global orientation function.

The last term simply represents elastic regularization. To enable minimization in thickness variance, we set $f_n(t) = \frac{\int^-_{\{p\in S\,|\,G_n(p)=t\}} D_n(\boldsymbol{p})dS}{\int^-_{\{p\in S\,|\,G_n(p)=t\}} dS}$. This may be thought of as a curve-based average thickness. Having obtained $\{f_n\}$, we minimize (7) according to the descent direction $\frac{dr_n}{d\tau} = \frac{2}{N}\left[f_n'\left(f_n - \bar{f}\right) - \sigma^2 r_n''\right]$, where $\bar{f}(t) = \frac{1}{N}\sum_{n=1}^{N} f_n[t - r_n(t)]$. We then adjust the GOF based on the 1-D maps by $G_{adj_n} = h_n^{-1} \circ G_n, h_n(t) = t - r_n(t)$.

4. SPHERICAL REGISTRATION

We modify an existing, highly robust framework for unconstrained spherical mapping proposed by Freidel et al. [8], by adding L^2 terms to the cost. Using Freidel's notation, the functional becomes $E = aE_{conformal} + bE_{area} + \sum_{i=1}^{K} \omega_i \int^-_{\mathbb{S}^2} \left(f_i(\boldsymbol{p}) - g_i(\boldsymbol{p})\right)^2 dS$, where f_i, g_i are scalar features of the moving and stationary spherical image. We use G_{adj} and D, the adjusted GOF and medial thickness features. Details on the optimization of this cost can be found in [8]. This is not a particularly innovative step, but a final surface-to-surface registration based on scalar features is needed to complete our shape matching. Our choice of registration framework is driven by practical concerns – alternative frameworks exist. In practice, this has proven to be a good choice. We use thickness as a feature in spherical registration, but thickness alone would not be sufficient to map the surfaces correctly, as it is a local measure. The global measure (GOF) primarily drives the registration.

5. IMPLEMENTATION AND EXPERIMENTS

We have applied our method to two datasets of left lateral ventricles: the ADNI baseline MRI dataset consisting of 391 subjects with mild cognitive impairment (MCI), and 229 age-matched controls; and a dataset of 11 HIV subjects and 8 age-matched controls [2]. Parameters were set according to **table 1** for both datasets. Our medial curve is evolved in a multi-resolution fashion with respect to both the curve and the surface: first, the surface is decimated to a small number of triangles (e.g., 300), and the curve is evolved at several resolution levels. We super-sample the curve by a pre-set ratio, as convergence is reached at the current level. The resulting curve is then used as initialization for the full mesh, at which point only a few descent steps are typically required. We show decrease in energy **(3)** with each step in **figure 3** for an HIV+ subject. Typical execution time is 2 minutes for a 10K triangle mesh on an AMD Opteron 152 2.61 GHz single core workstation with 4 Gb of RAM. An example of resulting curves and their GOFs can be seen in **figure 1**.

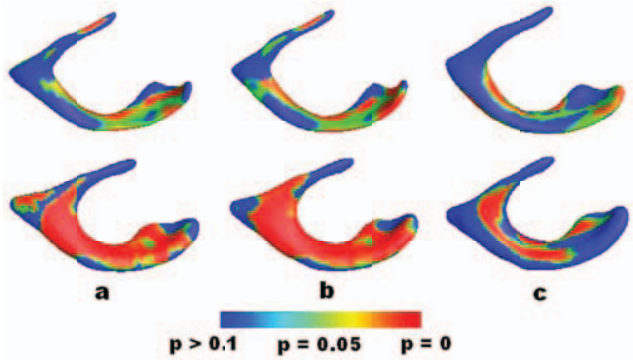

Figure 2. P-maps of HIV-NC (top row) and MCI-NC (bottom row) group difference after registering with (a) group-wise method, (b) unadjusted GOF + modified USP, (c) SPHARM

Group-wise registration reduced the variance of curve-based thickness by nearly 50% in the HIV cohort, and about 30% in ADNI. This step is computed in less than 1 minute. Our 1-D registration was done blindly with respect to both diagnosis and which subject was to be used as a target in spherical registration, to avoid "double-dipping". We show 3 average thickness functions from the ADNI dataset before and after the group-wise step in **figure 4**.

α	β	σ^2	ω_{thick}	ω_{GOF}	a	b
10	10^{-6}	0.1	10^2	10^6	5	2

Table 1. Parameters for group-wise shape registration

Our final spherical registration step was initialized by an unconstrained spherical map [8], followed by a spherical cross-correlation to remove the rotational component [3]. We then registered each subject to an arbitrary target shape from the same cohort using our modified USP framework. The L^2 energy was reduced by 20-80% depending on the subject and the cohort. Typical execution time was 10-20 seconds.

The statistical analysis consisted of computing overall p-value for group differences based on 100000 permutations, as in [4]. For comparison, we also applied the SPHARM method to the same two datasets. We then computed Cumulative Distribution Functions (CDF) of significance maps for each method (**fig. 5**). CDF curves are a way of visualizing the multiple comparisons problem, so that one sees the tradeoff between the statistical threshold and the spatial extent of the effect. While the permutation test remains the gold standard, CDF is still a useful visualization. As expected, group-wise registration improved the overall p-value in both studies, as seen in **table 2**. However, CDF plots show that using the unadjusted GOF, and keeping all other parameters the same produces the dominant curve.

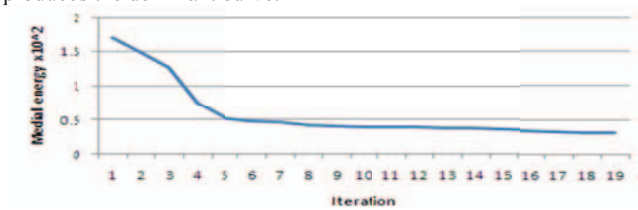

Figure 3. Medial energy **(3)** vs. gradient descent iterations

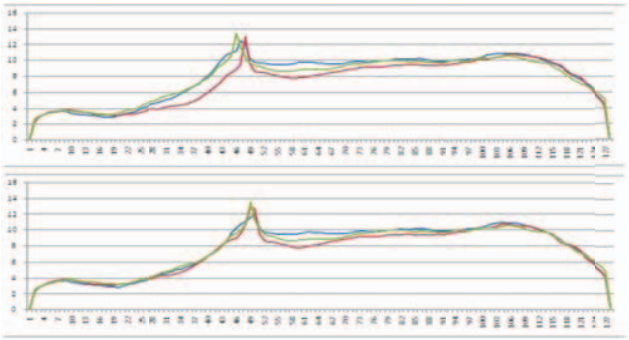

Figure 4. 1D Thickness maps of 3 ADNI subjects before (top) and after (bottom) group-wise registration of all 620 subjects. Note that the peaks corresponding to the posterior horn are well-matched.

This is expected: as group-wise registration actually localizes the effect, the resulting p-maps have smaller significant regions, with greater effect sizes. By focusing the significant regions, the group-wise method makes the p-maps more useful, as we see the true locations of significant change, rather than a washed out effect due to mis-registration. This effect is seen in **figure 2**

	Group	**No Group**	**SPHARM**
HIV	**0.00988**	0.01039	0.0149
ADNI	**0.00029**	0.00046	0.0068

Table 2. P-values for group difference after 100K perms

6. CONCLUSION

We presented a framework to register anatomical shapes that combines the description, registration, and statistical analysis aspects of shape comparison in one step. Our approach does not lead to true group-wise shape registration, but it approximates the solution to the group-wise problem by exploiting the specifics of the geometry of anatomical shapes - they are often approximately tubular. We further develop a medial curve method for computing intuitive descriptions of shapes so that the tubular assumption can be relaxed, and show that it works stably on the complicated lateral ventricles. As many anatomical shapes are inherently 1D, we reduce the computational problem significantly for group-wise registration. Thus, we are able to approximately group-wise register over 600 shapes in a few minutes.

We chose the most intuitive geometric measure, but other measures such as medial eccentricity, TV norm of the thickness, curvature-based features, functional and DTI-based measures, and others can all be incorporated easily into the framework. Future developments will include exploration of additional descriptors, as well as incorporating distance field information into the medial curve framework, and using a probabilistic atlas in the spherical domain.

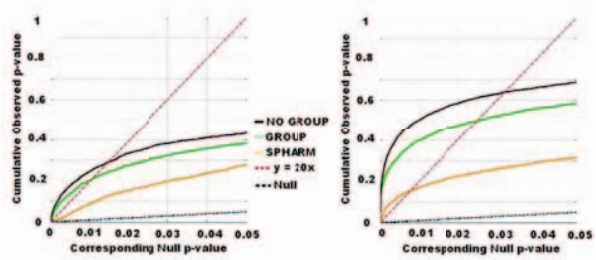

Figure 5. CDF plots of HIV vs. NC (left) and MCI vs. NC (right)

6. REFERENCES

[1]. P.A. Yushkevich , H. Zhang, JC Gee, "Parametric medial shape representation in 3-D via the Poisson partial differential equation with non-linear boundary conditions," *IPMI*. 19. pp.162-73. 2005.

[2] Y.L. Wang, J. Zhang, B. Gutman, T.F. Chan, J.T. Becker, H.J. Aizenstein, O.L. Lopez, R.J. Tamburo, A.W. Toga, P.M. Thompson, "Multivariate Tensor-Based Morphometry on Surfaces: Application to Mapping Ventricular Abnormalities in HIV/AIDS," *NeuroImage*, 49(3) pp. 2141-57, 2010.

[3] B. Gutman., Y. Wang, L.M. Lui, T.F. Chan, P.M. Thompson, A.W. Toga "Shape registration with spherical cross correlation," *MICCAI 2008 Workshop on Mathematical Foundations in Computational Anatomy (MFCA '08),* 2008.

[4] P.M. Thompson, K.M. Hayashi, G . de Zubicaray, A.L. Janke, S.E. Rose, J. Semple, M.S. Hong, D. Herman, D. Gravano, D.M. Doddrell, A.W. Toga, "Mapping Hippocampal and Ventricular Change in Alzheimer's Disease," *NeuroImage*, 22(4), pp. 1754-66, 2004.

[5]. S.M. Pizer, P.T. Fletcher, S. Joshi, A.G. Gash, J. Stough, A. Thall, G. Tracton, E.L. Chaney, "A method and software for segmentation of anatomic object ensembles by deformable m-reps." *Med Phys*. 32(5), pp. 1335-45, 2005.

[6] Y. Shi, J. Morra, P. M. Thompson, A.W. Toga " Inverse-consistent Surface Mapping with Laplace-Beltrami Eigen-features," *IPMI* 21, pp. 467–478 2009.

[7] R.H. Davies, C.J. Twinning, T.F. Cootes, J.C. Waterton, C.J. Taylor, "A Minimum Description Length Approach to Statistical Shape Modeling", *IEEE TMI* 21(5), pp. 525 – 538, 2002.

[8] I. Freidel, P. Schroeder, M. Desburn, "Unconstrained Spherical Parameterization," *J. Graphics, GPU and Game Tools* 12(1), pp. 17-26, 2007.

[9] N.D. Cornea, D. Silver, P. Min, "Curve-Skeleton Applications," *Proc. IEEE Visualization*, pp. 95-102, 2005.

[10] T. Yeo, et al., "Spherical Demons: Fast Diffeomorphic Landmark-Free Surface Registration", *IEEE TMI* 29(3), pp. 650 – 668, 2010.

[11] Brechbuhler, Ch, Gerig G, Kubler, O, "Parametrization of closed surfaces for 3D shape description." *Computer Vision and Image Understanding* 61(2), pp. 154–170, 1995.

[12] S. Kurtek, E. Klassen, Z. Ding, S.W. Jacobson, J.L. Jacobson, M.J. Avison, A. Srivastava, "Parameterization-Invariant Shape Comparisons of Anatomical Surfaces," *IEEE TMI* 30(3), pp. 849-858, 2011.