Abstract
We developed a general method for global surface conformal parameterization. For genus zero surface, we propose a new variational method which can find a unique conformal mapping between any two genus zero manifolds by minimizing the harmonic energy of the map. We apply this algorithm to the cortical surface matching problem. We further extended the algorithm to find a 3D volumetric harmonic map from a 3D brain volumetric model to a solid sphere. We use mesh structure to represent both surface and volumetric data. We proposed a new algorithm, sphere carving algorithm, which can build 3D tetrahedral mesh from a set of images with topology preservation. Experimental results on both synthetic data and real data are reported.

Conformal Mapping
- Any surface without holes or self-intersections can be mapped conformally onto the sphere
- This mapping, conformal equivalence, is one-to-one, onto, and angle preserving
- Locally, shape is preserved and distances and areas are only changed by a scaling factor
- A canonical space is useful for subsequent work

Genus Zero Surface Conformal Mapping Properties
- Intrinsic to geometry
- Independent of triangulation and resolution
- Depends on metric continuously
- Harmonic is equivalent to conformal
- All conformal are equivalent
- All the conformal construct a automorphism group: Möbius group which is a linear rational group on complex plane and a 6 dimensional group.

Surface Conformal Mapping Algorithm Details
- Minimize Harmonic Energy
- Use absolute derivative
- All computation are on the target surface, without projecting to complex plane
- Harmonic energy \( f: M \rightarrow S^2 \)
  \[ E(f) = \int f * dA \]
- Discrete harmonic energy \( E(f) = \sum_{i=1}^{n} |V_i| |
  \[ k_{wi} = \frac{1}{4}(\cot \alpha + \cot \beta) \]
- Discrete Laplacian
  \[ \Delta f(u) = \sum_{i=1}^{n} k_{wi}(f(u) - f(v)) \]

Volumetric Harmonic Map Algorithm Details
- Harmonic energy
  \[ E(f) = \sum_{i=1}^{n} (f(x_i) - f(y_i))^2 \]
- Minimize Harmonic Energy

Sphere Carving Algorithm
- Input (a sequence of volume images and a desired surface genus number)
- Output (a tetrahedral mesh whose surface has the desired genus number)
- Build a solid handle body tetrahedral mesh consisted of tetrahedra, such that the sphere totally enclose the 3D data.
- Let the boundary of the solid sphere be \( S \), We cut the model until we get the object 3D tetrahedral model.

Experimental Results

Harmonic Map
- The map minimizes the stretching energy
- Geodesics are harmonic maps from a circle to the surface
- In general, harmonic maps may not exist, or unique
- Depends on the Riemannian metrics. Independent of the embedding.
- For 3-manifold, the existence of harmonic maps, the uniqueness of harmonic maps, the diffeomorphic properties of harmonic maps are extremely difficult theoretical problems.
- Between convex 3-disk, harmonic map exists and is most likely diffeomorphism