Brain Surface Conformal Parameterization with Holomorphic Flow Method and Its Application to HIV/AIDS

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Introduction

We applied multivariate tensor-based morphometry to study lateral ventricular surface abnormalities associated with HIV/AIDS.
Based on holomorphic one-forms, we obtained conformal parameterization of ventricular geometry and registered lateral ventricular surfaces across 19 subjects (8 controls and 11 HIV/AIDS). Multivariate Hotelling's T^2 statistic was applied on the local Riemannian metric tensors, and it powerfully detected brain surface abnormalities.

Methods

1. Canonical Conformal Parameterization

Holomorphic one-forms, a structure used in differential geometry, can be used to generate canonical conformal parameterization:

- Compute exact harmonic one-forms bases;
- Compute closed harmonic one-forms bases;
- Compute holomorphic one-forms bases;
- Compute the canonical holomorphic one-form and the canonical conformal parameterization.

2. Multivariate Tensor-Based Morphometry

Suppose $\phi: S_1 \rightarrow S_2$ is a map from the surface S_1 to the surface S_2, and the derivative map of ϕ is the linear map between the tangent spaces:

$$d\phi: TM(p) \rightarrow TM(\phi(p))$$

Under the orthonaomal frame on S_1 and S_2:

$$\{e^{-\lambda_1}\frac{\partial}{\partial u_1}, e^{-\lambda_1}\frac{\partial}{\partial v_1}\}, \{e^{-\lambda_2}\frac{\partial}{\partial u_2}, e^{-\lambda_2}\frac{\partial}{\partial v_2}\}$$

Then, the derivative map under the orthonomal frames is represented by :

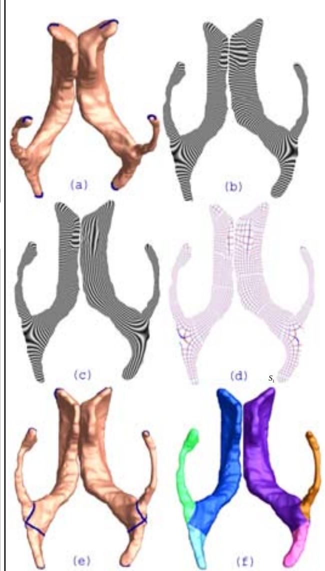
$$J = d\phi = e^{\lambda_1-\lambda_2}\begin{pmatrix} \frac{\partial\phi_1}{\partial u_1} & \frac{\partial\phi_1}{\partial v_1} \\ \frac{\partial\phi_2}{\partial u_1} & \frac{\partial\phi_2}{\partial v_1} \end{pmatrix}$$

Define the deformation tensor as: $S = (J^T J)^{1/2}$, and transform it via a logarithmic transformation to form a vector space.
We applied hotelling's T^2 statistic on Mahalanobis distance :

$$M = (\log \overline{S} - \log \overline{T})\Sigma^{-1}(\log \overline{S} - \log \overline{T})$$

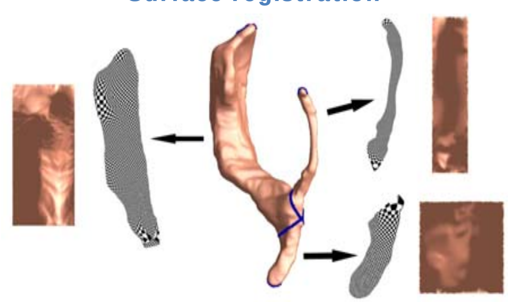
and performed non-parametric permutation test.

Surface parameterization and partition



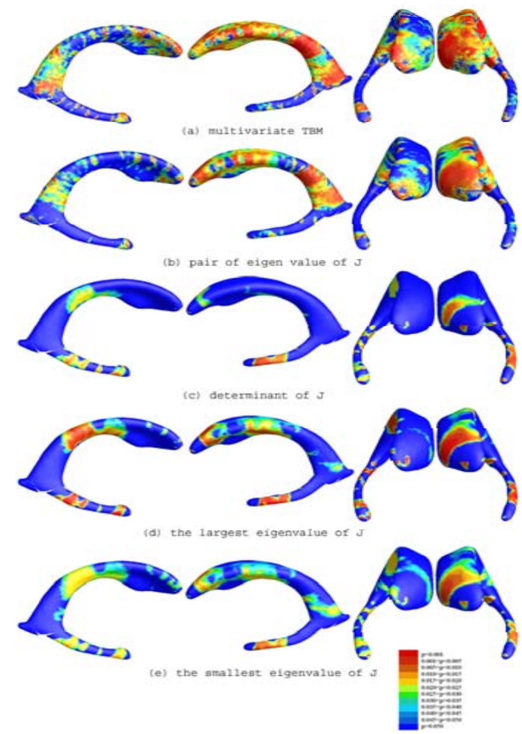
•Three cuts were automatically located and introduced for topology optimization. After modeling the topology in this way, it became an open boundary surface with 3 boundaries(Fig.(a)).
•The exact harmonic one-form(Fig.(b)), its conjugate one-form (Fig.(c)) and canonical holomorphic one-form (Fig.(d)) were then computed.
•According to the conformal net (Fig.(e)), each surface can be divided into three connected pieces (Fig.(f)), with each piece topologically equivalent to a cylinder.

Surface registration



After segmentation, a new canonical holomorphic one-form is computed on each piece and each piece is conformally mapped to a rectangle. Surface registration is performed via parameter domain.

Statistical verification



After analyzing various different surfaced-based statistics, the permutation test p values were calculated for each surface point, and the result is shown above. The overall significance levels were also computed, with the smallest to be 0.0028 for the left ventricle and 0.0066 for the right, resulting from multivariate TBM.

Comparison with Other Statistics

	Full Matrix J	Determinant of J	Largest EV of J	Smallest EV of J	Pair of EV of J
Left Vent	**0.0028**	0.0330	0.0098	0.0240	0.0084
Right Vent	**0.0066**	0.0448	0.0120	0.0306	0.0226

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