# Shape Analysis with Conformal Invariants for Multiply Connected Domains and its Application to Analyzing Brain Morphology

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## Abstract

•We propose to compute a novel shape analysis method based on conformal invariant (CI), which is associated with the perimeter of the inner concentric circle in the hyperbolic parameter plane.

•Our algorithm provides a stable method to compute the new shape index in the 2D (hyperbolic) parameter domain by Ricci Flow method.

•We applied it for analyzing abnormalities in brain morphology in Alzheimer's disease (AD) and Williams syndrome (WS).

## Benefits

•CIs are robust to rigid-body transformations and conformal transformations of the original surfaces.

• Local geometry is well preserved under conformal mapping, so CIs are good candidate features for brain research on cortical and subcortical surface morphology.

## Methods

• Get multiple connected domains by cutting along landmark curves on surfaces of the cerebral cortex or hippocampus,

• Conformally project the surfaces to hyperbolic plane with surface Ricci flow method

• Compute the proposed CI for each landmark curve, and assembled these into a feature vector,

• Detect group differences in brain structure based on multivariate analysis of the surface deformation tensors.

## Conclusions

• We propose a stable way to compute a conformal invariant (a vector-valued shape index) for a multiply connected domain by Ricci Flow method.

• It can reliably detect systematic morphometric abnormalities for brains, at the group level, between control subjects and patients.

• It can determine the surface template for registration, with provably better power for detecting abnormalities by group study.

## Conformal Invariant

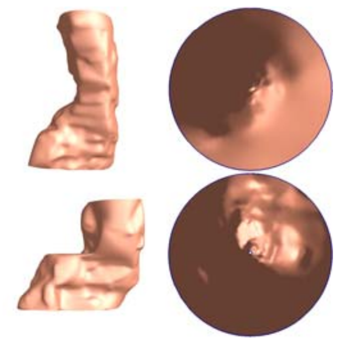

Figure 1. Conformal invariant for hippocampal surfaces (two holes on the front and back: the anterior junction with the amygdala and the posterior limit as it turns into the white matter of the fornix)
Row1: from a control subject, CI=0.009;
Row2: from an AD patient, CI=0.123.

## Vector-valued Shape Index

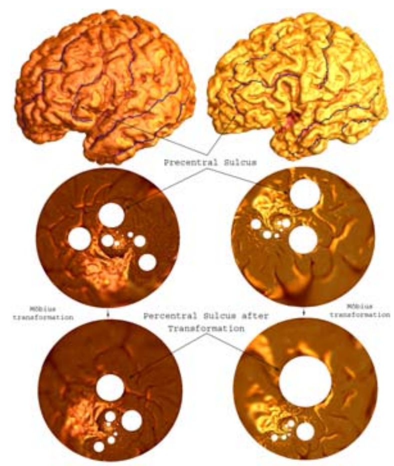


Figure 2. Conformal invariant for the Precentral Sulcus of the left hemisphere, where it is mapped to the center circle.
Col1: for a control subject, CI=0.472;
Col2: for a WS patient, CI=1.513.

## Surface Templates for Group Study

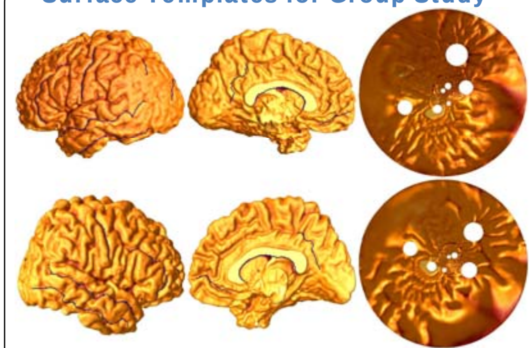

Figure 3. Illustration of selected surface templates for left (Row1) and right (Row2) hemispheres, for our multivariate statistical study of group differences in cortical morphometry. By computing the Mahalanobis distance from each individual cortical surface to each of the two groups, the ones with smallest sum of the two distances were selected. Their parameterization results were used as the template meshes for registration.

## Statistically Significant Differences

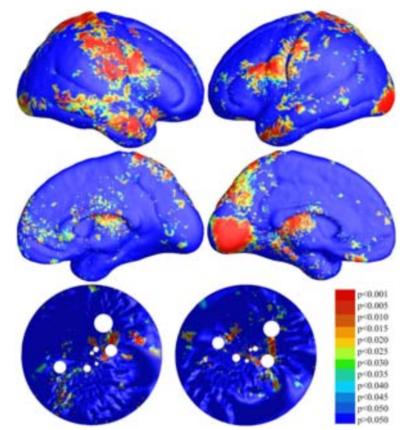

Figure 4. Map of statistically significant differences in cortical morphometry between 40 controls and 40 WS subjects. Non-blue colors denote the area with a significant statistical difference p=0.05. The overall permutation significance *p* values are 0.001 and 0.0006 for right and left hemisphere, respectively.

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