Cortical Surface Classification with Hyperbolic Wasserstein Distance

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Introduction

The Wasserstein space consists of all probability measures that are defined on a Riemannian manifold. The Wasserstein distance defines a Riemannian metric for the Wasserstein space and it intrinsically measures the similarities between shapes. Wasserstein distance has been widely studied and applied in image and shape analysis. However, existing methods for Wasserstein distance computation only work with 2D images or genus-0 surfaces. This work proposes a novel algorithm to compute Wasserstein distance between general topological surfaces with hyperbolic metric, which is called hyperbolic Wasserstein distance. We applied it to cortical surface classification in Alzheimer's disease (AD).

Methods

- An MR image is automatically segmented and cortical surfaces are reconstructed with the FreeSurfer software [1].
- Six landmark curves are automatically traced on each cortical surface using the Caret package [2], as shown in Fig. 1. By slicing each cortical surface along the landmarks, we model it as a genus-0 surface with 6 boundaries.
- We compute the hyperbolic metric of each cortical surface, which is conformal to its original Euclidean metric, with the hyperbolic Ricci flow [3], and isometrically embed it onto the Poincaré disk, as shown in Fig. 2 (a).
- With deck transformation group generators [4], we tile a finite portion of the universal covering space of each cortical surface, as shown in Fig. 2 (b). Then we obtain its canonical Poincaré disk domain using the geodesic curve lifting algorithm [4], as shown in Fig. 2 (c).
- We covert the Poincaré disk to Klein model and use the Klein disk as a canonical space to establish the initial mapping between two cortical surfaces [4], as shown in Fig. 2 (d).
- The initial mapping is diffused to a global hyperbolic harmonic map between two surfaces [5], as shown in Fig. 2 (e).
- We compute the surface tensor-based morphometry (TBM) [4] of the hyperbolic harmonic map as a probability measure on each cortical surface.
- Using the Poincaré disk as a canonical space, we compute the optimal mass transportation (OMT) map [6] between two measures with the power Voronoi diagram [7], which has been extended to hyperbolic space, as shown in Fig. 2 (f).
- The transportation cost of the hyperbolic OMT map defines the proposed hyperbolic Wasserstein distance between two cortical surfaces.

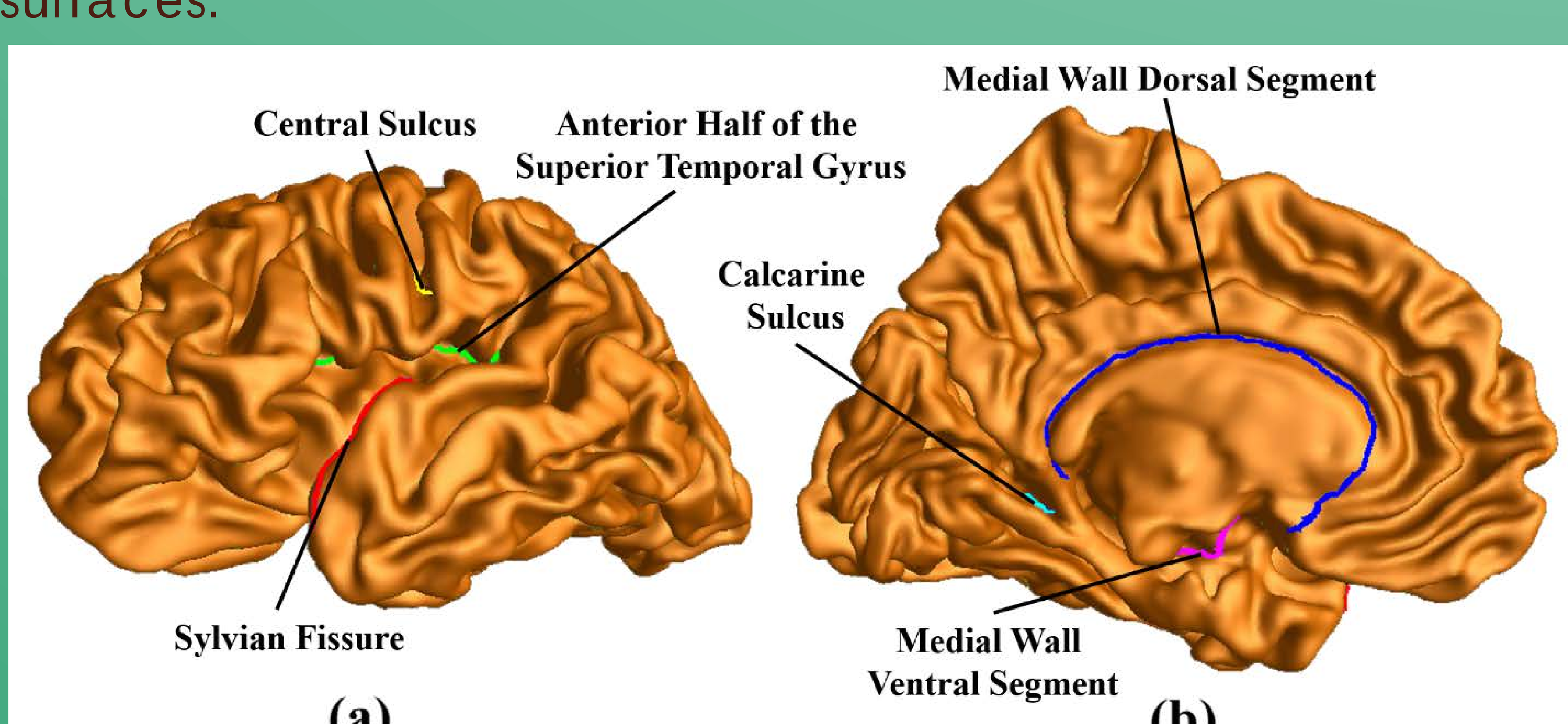

Figure 1. Landmark curves on a left cortical surface, which are automatically labeled by Caret [2], shown in two views.

Experiments

- *Longitudinal Cortical Morphometry*. We randomly selected a healthy subject (85-year male) from the Alzheimer's Disease Neuroimaging Initiative (ADNI) dataset. Longitudinal structural MRIs at 3 time points were studied, the baseline, 12 months and 24 months. The baseline cortical surface was used as the template of the hyperbolic harmonic map and OMT map. The resulting hyperbolic power Voronoi diagrams of the 12-month and 24-month surfaces are shown in Fig. 3 and their hyperbolic Wasserstein distances are 132.28 and 201.70, respectively. This shows that our method may provide an imaging index to study the longitudinal brain morphometry.

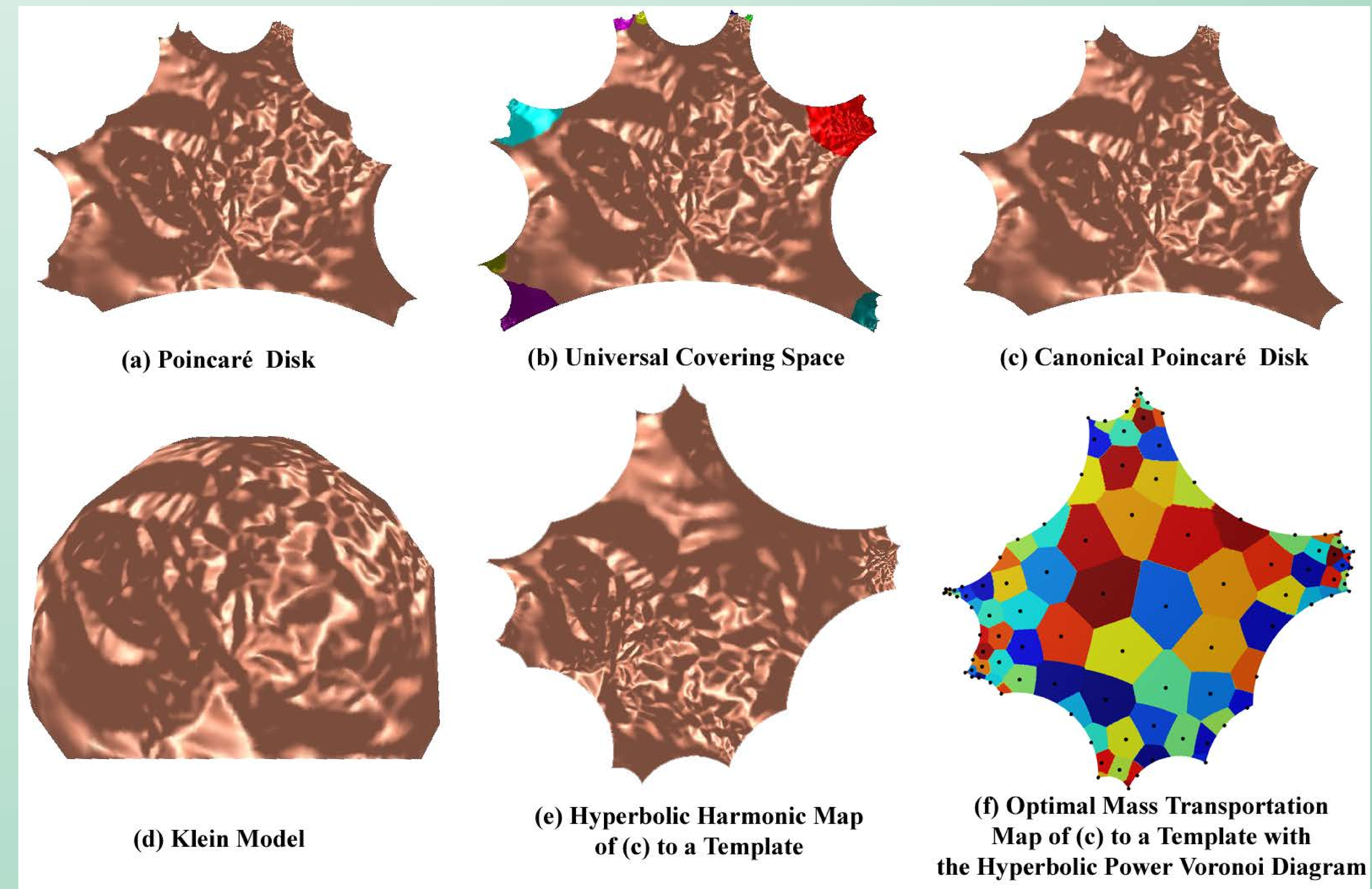

Figure 2. System pipeline overview.

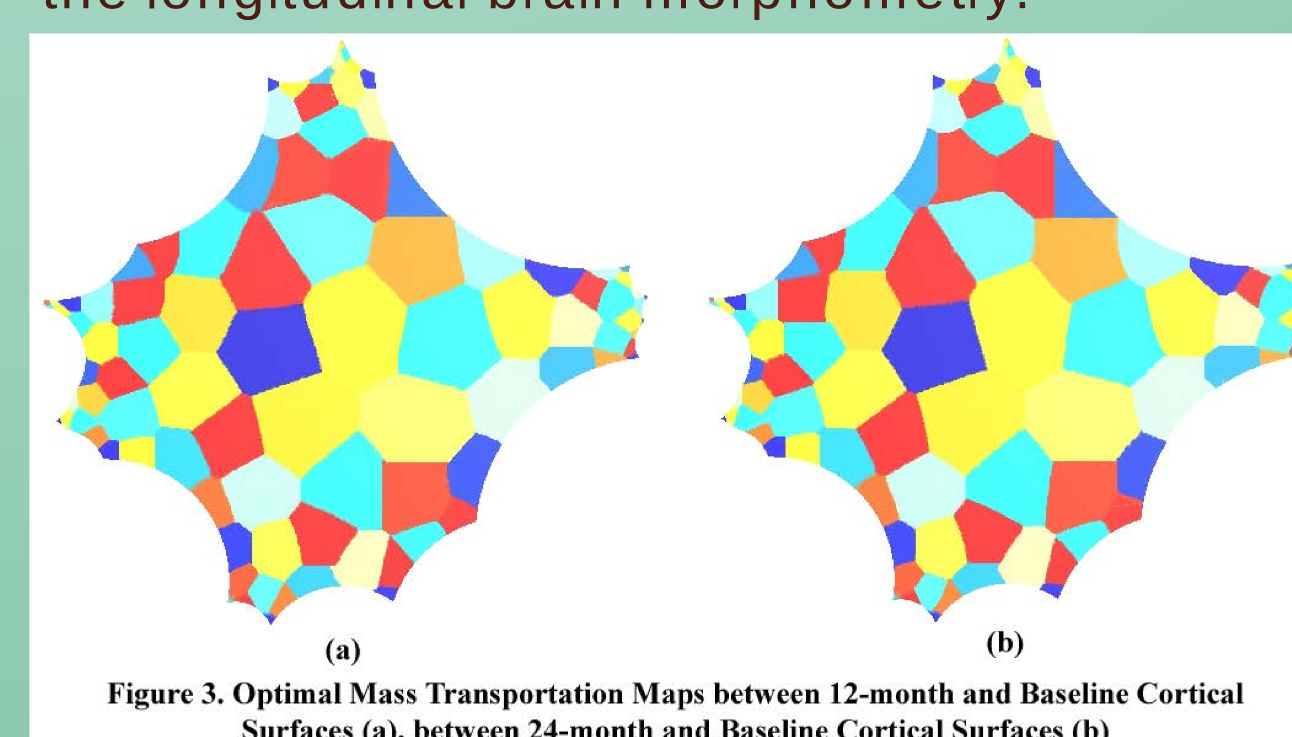

Figure 3. OMT maps between 12-month and baseline cortical surfaces (a), between 24-month and baseline cortical surfaces (b).

- *Cortical Surface Classification*. We randomly selected 30 AD patients and 30 healthy people from the ADNI baseline dataset. A healthy subject, which is not in the 60 studied group, was used as the template. We applied the complex tree classifier in MATLAB. With a 5-fold cross validation, the classification rate of our method is 76.7%. We also compared with two other standard surface features: surface volume and area. The comparison result is shown in Fig. 4.

Method	Classification Rate
hyperbolic Wasserstein distance	76.7%
Surface Area	41.7%
Surface Volume	51.7%

Figure 4. Classification rate comparison of our method and two other cortical surface shape features: surface are and volume.

Citations

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