Conformal Invariants for Shape Analysis in Brain Morphometry Study

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Introduction
Shape space has been widely studied in computer vision and medical imaging. Two surfaces are conformally equivalent if they can be mapped to each other with a conformal map. All conformally equivalent surfaces form the Teichmüller space. The coordinate of a surface in the Teichmüller space is invariant under conformal maps, so it may provide a simple and refined index to represent a unique shape. In brain imaging field, landmark curves are very important for shape correspondence computation and quantitative brain morphometry analysis. This work proposes a novel method to compute conformal invariants for cortical surfaces with multiple (more than 2) landmark curves and applies them to analyze abnormalities in brain morphometry associated with Alzheimer’s disease (AD).

Methods
- Given an MR image, we segment it and reconstruct the cortical surfaces with the FreeSurfer software package [1].
- We automatically label six landmark curves on each cortical surface using the Caret tool [2]. The landmarks include Central Sulcus, Anterior Half of the Superior Temporal Gyrus, Sylvian Fissure, Calcarine Sulcus, Medial Wall Ventral Segment, and Medial Wall Dorsal Segment, as shown in Fig. 1.
- With topology optimization [3], we model the landmarks as open boundaries. The resulting cortical surfaces admit hyperbolic geometry and their hyperbolic uniformization metric, which is conformal to the original Euclidean metric, can be computed with the hyperbolic Ricci flow algorithm [4]. With the resulting hyperbolic metric, we can conformally map the surface to the 2D Poincaré disk, as shown in Fig. 2.
- The conformal invariants of each cortical surface are defined as lengths of the six boundaries in the hyperbolic Poincaré disk.

Experiments
- Longitudinal Cortical Morphometry in AD. We randomly selected two mild cognitive impairment (MCI) patients from the Alzheimer’s Disease Neuroimaging Initiative (ADNI) database. One MCI patient who converted to AD 36 months after the baseline screening is called the MCI converter subject, the other who did not convert to AD during the same period is called the MCI stable subject. The structural MR images at two time points were studied, the baseline and 24 months. The hypothesis is that we may observe some different atrophy patterns associated with the conversion vs. stable progression paths. With the proposed method, the difference between the conformal invariants at two time points is 0.3925 and 1.0438 for the MCI stable and converter subjects, respectively. To illustrate the difference on each conformal invariant, we also plotted the radar chart [5], as shown in Fig. 3.

• Multivariate Statistics on Population-based Group Difference Study. In this experiment, we applied a group-wise statistical analysis to the proposed conformal invariants in left hemisphere cortex between AD and control subjects. Our dataset consisted of 60 subjects which were randomly chosen from ADNI baseline database, including 30 AD subjects and 30 matching normal controls. The demographic information of these subjects is shown in Table 1. We used Hotelling’s $T^2$ [6] test to estimate the difference between the two groups and used permutation test [3] to evaluate the overall significance of the group comparison. With the proposed method, the significant $p$-value of the group comparison is 0.0133. We also compared with two other standard surface features, surface area ($p$-value=0.5888) and volume ($p$-value=0.1152).

<table>
<thead>
<tr>
<th>Gender(M/F)</th>
<th>Education</th>
<th>Age (±3.73)</th>
<th>MMSE at Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTL</td>
<td>1/17</td>
<td>15.1 ± 2.1</td>
<td>79.73 ± 7.39</td>
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<tr>
<td>AD</td>
<td>14/16</td>
<td>15.4 ± 2.8</td>
<td>82.52 ± 6.12</td>
</tr>
</tbody>
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Table 1. Demographic information of 60 studied subjects in ADNI baseline dataset.

Citations

Figure 1. Landmark curves on a left cortical surface, which are automatically labeled by Caret [2], showing with two different views.

Figure 2. A cortical surface with six landmark curves, which are denoted as $y_i$ (a) and its conformal map to the Poincaré disk (b).

Figure 3. Radar chart on individual landmark conformal invariants between two MCI subjects. Each landmark is associated with a corner on a hexagon and six axes are connected to the common origin point. The position on the axis is proportional to the absolute difference between two landmark conformal invariants computed on different time points. By connecting the six points from the same subject, we get two new hexagons. The blue one stands for MCI Stable (MCI-Sta) and the orange for MCI converter (MCI-Con). The radar chart provides a simple way to display multivariate data in the form of a two-dimensional chart.