Brain Surface Conformal Slit Mapping

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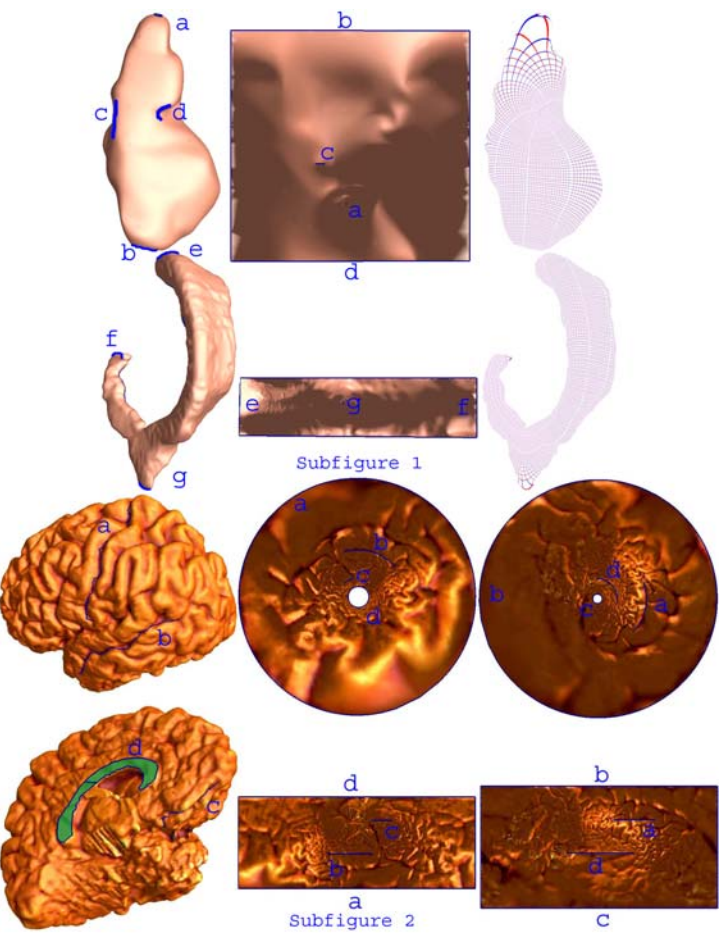
Objective: We propose a method that computes a conformal mapping from a multiply connected mesh to the so-called *slit domain*, which consists of a canonical rectangle or disk in which 3D curved landmarks on the original surfaces are mapped to concentric or parallel lines in the slit domain. The whole algorithm is based on solving linear systems so it is very stable. In the slit domain parameterization results, the feature landmark curves on the surface are either mapped to straight lines or a concentric arcs. This representation is convenient for anatomical visualization, and may assist statistical comparisons of anatomy, surface-based registration and signal processing.

Algorithm Pipeline: Suppose the input mesh has n+1 boundaries, $\partial M = \gamma_0 - \gamma_1 - \cdots - \gamma_n$ Without loss of generality, we map γ_0 to the outer circle of the circular slit domain, γ_1 to the inner circle, and all the others to the concentric slits.

The algorithm pipeline is as follows:

1. Compute the basis for all exact harmonic one-forms;
2. Compute the basis for all closed harmonic one-forms;
3. Compute the basis for all holomorphic one-forms;
4. Construct the slit mapping

Figure 1(on the right). (a) and (b) show the cortical surface with 12 landmarks cut open, including an open boundary at the corpus callosum (in green); (c) is the parallel slit map result; (f) is the circular slit map result; (d) and (e) show the conformaltexture parameterized by the circular slit map (f).



Results

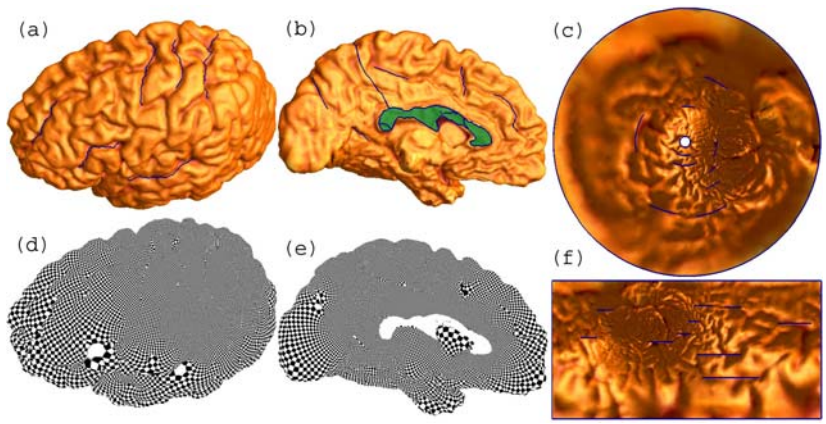


Figure 2 (on the left). Subfigure 1 illustrates the parallel slit mapping results for a hippocampal surface and for a surface of the left lateral ventricle.
In the first row, four landmarks are cut open on a hippocampal surface. In the parallel slit map result, landmarks b and d are mapped to the upper and lower boundaries, respectively. On the second row, three landmarks are introduced on a lateral ventricle surface. In the parallel slit map, e and f are mapped to the left and right boundaries, respectively. Their conformal texture are also shown. In Subfigure 2, conformal parameterization results are shown with different boundary conditions. The first column shows a cerebral cortical surface with 4 landmarks introduced as cuts. The second column shows the circular slit map and parallel slit map results when a pair of landmarks are selected as boundaries landmark a as the exterior circular boundary and d as the inner circular boundary.
The third column shows results the other pair of landmarks are selected as boundaries.

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